Numerical Integration

Example 1. Compute

$$\int_0^2 \sin(x)\,dx = 1 - \cos(2) = 1.41614683654714$$

using the left endpoint Riemann sum and the Trapezoid method, using different numbers of points n. We obtain the following results:

n	left sum	trapezoid
10	1.32049345441631	1.41142319709888
50	1.39777206339704	1.41595801193356
100	1.40700665706963	1.41609963133789
500	1.41432635349721	1.41614494835086
1000	1.41523706707134	1.41614636449817

The correct digits are underlined in each case.

Example 2 Compute the distance the point P travels in Example 4.1 on page 20 of the notes, up to the time when the wheel hits the ground.

The location of point P at time t is given by

$$\begin{aligned} x(t) &= 6t + 0.2\sin(30t) \\ y(t) &= 122.7 - 4.9t^2 + 0.2\cos(30t) \end{aligned}$$

The wheel hits the ground when the center is at $y = 0.2$, which occurs at $t = 5$.

The distance P travels is

$$\begin{aligned} \text{distance} &= \int_0^5 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt \\ &= \int_0^5 \sqrt{(6 + 6\cos(30t))^2 + (-9.8t - 6\sin(30t))^2}\, dt \end{aligned}$$

We can approximate this integral numerically using the trapezoid rule and we get the following approximations for different values of n:

n	trapezoid
100	131.077054300453
1000	131.034411973093
10000	131.034274215187
100000	131.034270889627