Assignment 6.

Due Wednesday, June 4 **at start of class**.

Objectives: To prove some results about stationary iterative methods and steepest descent; to use stationary iterative methods and the conjugate gradient method to solve the linear system from the steady-state heat equation in two dimensions.

(1) pp. 229–230, problems 1 and 20.

(2) p. 244, problem 1.

(3) On the course web page: `www.math.washington.edu/~greenbau/Math_466/page.html` is a finite difference code (steady2d.m) to solve the boundary value problem:

$$\frac{\partial}{\partial x}\left(a(x,y)\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(a(x,y)\frac{\partial u}{\partial y}\right) = f(x,y) \quad \text{in } (0,1)\times(0,1)$$

$$u(x,0) = u(x,1) = u(0,y) = u(1,y) = 0,$$

where $a(x,y) = 1+x^2+y^2$ and $f(x,y) = 1$. It uses a direct solver for the linear system.

Replace this direct solver first by the Jacobi method, then by the Gauss Seidel method, and then by the SOR method. For each method, make a plot of the relative residual norm, $\|b - Au^k\|/\|b\|$ versus iteration number k. Try several different values for the parameter ω in SOR, until you find one that seems to work well.

Then try solving the linear system using the conjugate gradient method. You may write your own CG code or use the one in Matlab (called **pcg**). You may use a preconditioner with your CG routine or just take the preconditioner to be the identity. Again make a plot of relative residual norm versus iteration number for the CG method.

Experiment with a few different mesh sizes and comment on how the number of iterations required to reach a fixed level of accuracy seems to vary with h for each method.