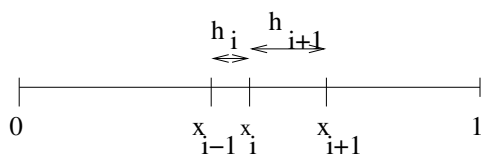


## Assignment 4.

Due Friday, Apr. 27.

- (1) Suppose you wish to solve the two-point boundary value problem  $u''(x) = f(x)$ ,  $0 \leq x \leq 1$ ,  $u(0) = \alpha$ ,  $u(1) = \beta$ , using a *nonuniform* grid with spacing  $h_i$  between  $x_{i-1}$  and  $x_i$ ,  $i = 1, \dots, n$ :



- (a) Write down an approximation to  $u''(x_i)$  in terms of  $u(x_{i-1})$ ,  $u(x_i)$ , and  $u(x_{i+1})$ :  $u''(x_i) \approx au(x_{i-1}) + bu(x_i) + cu(x_{i+1})$ . Choose the coefficients  $a$ ,  $b$ , and  $c$  to make the order of accuracy as high as possible and determine what this order of accuracy is.
- (b) EXTRA CREDIT: Find a way to approximate  $u''(x_i)$  that is *second order* accurate on a nonuniform grid. Write the resulting difference equations for the two-point boundary value problem in matrix form; i.e., say what the entries of  $A$  are when the equations are written in the form  $A\mathbf{u} = \mathbf{b}$ , where  $A$  is an  $n - 1$  by  $n - 1$  matrix and  $\mathbf{u}$  is the vector of unknown values at the interior nodes.
- (2) Problems 1 and 2 on p. 275.