Assignment 7. Due Friday, Mar. 6.

Reading: Course Notes, finish chapter on Lebesgue integration,
Jones, chs. 2, 5, 6, 8.

1. Recall that $L^p(\mathbf{R}^n)$ consists of equivalence classes of functions equal a.e.

   (a) Show that any such equivalence class can contain at most one continuous function (so it makes sense to say that an element of $L^p(\mathbf{R}^n)$ is continuous.)

   (b) Show that if $f \in L^\infty(\mathbf{R}^n)$ and f is continuous, then $\|f\|_\infty = \sup_{x\in\mathbf{R}^n} |f(x)|$.

2. Find a continuous function f on $\mathbf{R}$ so that $f \in L^1(\mathbf{R})$ but $f \notin L^\infty(\mathbf{R})$.

3. True or False (Give proof or counterexample). Suppose $x_n, y_n, x, y \in H$, a Hilbert space.

   (a) If $x_n \to x$ weakly and $y_n \to y$ strongly, then $\langle x_n, y_n\rangle \to \langle x, y\rangle$.

   (b) If $x_n \to x$ weakly and $y_n \to y$ weakly, then $\langle x_n, y_n\rangle \to \langle x, y\rangle$.