Assignment 5. Due Fri., May 20.

Reading: Read section of the Notes on "Sturm-Liouville Problems"
Read chapters 1 and 2 in Friedlander.

1. Do problems 1.4, 1.5, 1.9, and 2.1 in Friedlander.

2. Let

$$\partial_t u = P(\partial_x)u, \quad x \in \mathbf{R}^n,\ t \geq 0$$
$$u(x,0) = u_0(x), \quad x \in \mathbf{R}^n$$

be a Cauchy problem with constant coefficients which is well-posed in L^2; here $u(x,t) \in \mathbf{C}^s$ and $P(\partial_x) = \sum_{|\alpha| \leq m} A_\alpha \partial_x^\alpha$, where each $A_\alpha \in \mathbf{C}^{s\times s}$. Show that if u is a "generalized solution" of the differential equation, as defined in the Notes on p. 2 of the section on the Cauchy Problem, then if we view u as a distribution on $\mathbf{R}^n \times (0,\infty)$, then u satisfies $\partial_t u = P(\partial_x)u$ in the sense of distributions. Justify your steps carefully.