Some Possible Project Topics:

1. Consider an application area that you are interested in and where you think that nonnormality might have a significant effect on the matrices or operators involved. Investigate what can be learned from the spectrum and also what can be learned by looking at other matrix/operator properties such as the field of values, pseudospectra, and polynomial numerical hulls.

2. Report on a chapter or section(s) of the text that will not be covered in class. (The ones that I plan to cover are listed in the syllabus; if you want to do one of these instead, let me know and I can modify the course plan.)

   Some chapters that look interesting to me and that I do not plan to cover in class include:

   - Ch. II, Toeplitz Matrices.

     What is known about pseudospectra and polynomial numerical hulls of Toeplitz matrices and/or operators, and what does this tell us about the behavior of such operators?

   - Ch. III, Differential Operators, especially sec. 13.

     Some PDE's have no solutions even locally. This is related to exponential growth of the resolvent norm. [Note: This section is likely to be very mathematical, as opposed to applied or computational.]

   - Ch. V, Fluid Mechanics.
   - Ch. VII, Numerical Solution of Differential Equations.

     A standard way of analyzing stability of difference equations is to look at the test problem $y' = \lambda y$ and determine what values of the timestep h will yield stable difference equations for this problem. But this is an eigenvalue analysis. Are there better ways to analyze stability of difference equations?

   - Ch. VIII, Random Matrices.
   - Ch. X, sec. 56–57, Markov Chains.

     Eigenvalues are not necessarily the right tool for describing the behavior of Markov chains. What can pseudospectra or polynomial numerical hulls tell us that eigenvalues cannot?

   - Ch. X, sec. 58, Population Ecology.

3. Report on a paper from the literature. (You might want to do this in conjunction with one of the chapters or sections in the text.)

4. If you have some ideas about an open problem and would like to pursue them, that would be fine. Do not necessarily expect to solve the problem, however.

   For example, you might consider Crouzeix's conjecture ($\|p(A)\| \leq 2 \max_{z \in \mathcal{F}(A)} |p(z)|$ for all polynomials p). You could look at his paper (available on his web page; uses a lot of complex analysis), and you might write a code to try and test the conjecture numerically and perhaps get a better idea about how one might prove/disprove it.

   If you have some ideas about better ways to compute pseudospectra or polynomial numerical hulls, you could write a code to do this. The text does not seem to say much about the accuracy of pseudospectral computations. If this question is of interest, you could try to develop an error analysis.

   Other open problems are likely to be mentioned as the course continues.