1. **(10pts.)** Let $D$ be the region inside the box $\{(x,y) : -3 \leq x \leq 0, -2 \leq y \leq 0\}$ but outside the circle $\{(x,y) : x^2 + y^2 = 1\}$ (see picture). Compute $\displaystyle\int\int_D xy dA$.

   We can split $D$ into two separate non-intersecting regions, with a vertical line at $x = -1$. On the left, we have a rectangle $D_1$, $D_1 = \{(x,y) : -3 \leq x \leq -1, -2 \leq y \leq 0\}$. On the right, we have a region $D_2$ bounded above by the circle, $D_2 = \{(x,y) : -1 \leq x \leq 0, -2 \leq y \leq -\sqrt{1-x^2}\}$.

$$\int\int_D xy dA = \int\int_{D_1} xy dA + \int\int_{D_2} xy dA = \int_{-3}^{-1}\int_{-2}^{0} xy dy dx + \int_{-1}^{0}\int_{-2}^{-\sqrt{1-x^2}} xy dy dx = 8 + \frac{7}{8}$$

2. **(5pts.)** A flying saucer is hovering overhead. Find the mass of the saucer if it occupies the space bounded below by the hyperboloid $z = \sqrt{x^2 + y^2}$ and above by the plane $z = 2$, with a density given by the function $\rho(x, y, z) = 6z$. (You need not worry about units; but if it matters to you, $x$, $y$, and $z$ are all given in rods, and $\rho$ is in slugs per cubic rod.)

   Mass is the integral over the volume of density; we wish to set this up as an iterated integral, so it seems reasonable to try to integrate with respect to $z$ first. We're given that $\sqrt{x^2 + y^2} \le z \le 2$, so the only restriction on $x$ and $y$ is that $\sqrt{x^2 + y^2} \le 2$, or $x^2 + y^2 \le 4$: this is the disk of radius 2 centered at the origin in the $xy$-plane. Call this disk $D$. Then...

$$m = \int\int\int 6z dV = \int\int_D \int_{\sqrt{x^2+y^2}}^{2} 6z dz dA = 3 \int\int_D 4 - x^2 - y^2 dA$$

   To make life easier, convert things into polar coordinates now.

$$m = 3 \int_0^{2\pi} \int_0^2 (4 - r^2) r dr d\theta = 24\pi$$

3. **(5pts.)** Now find the $z$-coordinate of the center of mass of the saucer.

   We integrate density times $z$, and then divide by the mass $m$.

$$\bar{z} = \frac{1}{m}\int\int\int 6z^2 dV = \frac{1}{m}\int\int_D \int_{\sqrt{x^2+y^2}}^{2} 6z^2 dz dA = \frac{2}{m}\int\int_D 8 - (x^2+y^2)^{3/2} dA$$

   Again we convert to polar coordinates, remembering that $x^2 + y^2 = r^2$:

$$\bar{z} = \frac{2}{m}\int_0^{2\pi}\int_0^2 (8 - r^3) r dr d\theta = \frac{8}{5}$$