Math 425 – Final Exam Study Topics (includes the Midterm topics)

General principles:

• You may be asked to prove a result that is similar to a proof in the book. Write the proof; don't just say "it's like a proof in the book".

• Unless explicitly told otherwise, you may use any theorem (or lemma or corollary) from the sections of the text below, as well as results proved in Math 424. If you use a theorem, state clearly what you are using, like "By a theorem in the book we know that every continuous function on a compact set is bounded"; you don't need to give the number of the theorem.

• Do not use results from homework problems (even if they were assigned) unless you reprove them.

Section 3.2: Riemann/Darboux integration: know the definition of the Darboux integral, $U(f, P)$ and $L(f, P)$; Riemann-Lebesgue Lemma.

Section 3.3: Series: notion of convergence of series and absolute convergence. Ratio/root tests; comparison test; integral test.

Section 4.1: Uniform convergence and $C_b(M)$ and $C^0([a, b])$. Pointwise verse uniform convergence; $C_b(M)$ is complete in the uniform norm; series of functions and the Weirstrass M-test; integration and limits; differentiation and limits.

Section 4.2: Power series: radius of convergence; uniform convergence of series; differentiation and integration of power series.

Section 4.3: Equicontinuity and Arzela-Ascoli. Compactness of subsets of $C^0([a, b])$ under uniform norm.

Section 4.4: Weirstrass approximation theorem for polynomials in $C^0([a, b])$; know the statement, questions would be about applications.

Section 4.5: Contractions and ODE's. Know the Banach contraction principle: assumptions and conclusion, and idea of the proof. For ODE's, know the definition of systems, the assumptions and conclusion of the Picard Theorem.

Section 5.1: Vector spaces and norms. Know the definition of a norm, how to verify if something is a norm. For linear maps bounded=continous; know what it means to show a linear map is bounded.

Section 5.2: Derivatives of functions in higher dimensions. Definition of differentiability and derivatives of maps $f : \mathbb{R}^m \to \mathbb{R}$ and $f : \mathbb{R}^m \to \mathbb{R}^n$. Continuity of partial derivatives implies differentiability. Chain rule.

Section 5.4: Inverse and Implicit Function theorems. Know the statements (assumptions and conclusions).