Math 425 Autumn 2018

Homework 2 **Due Monday, January 22**

Chapter 3 [Pugh, pg. 198]: 51, 53, 62.

Additional problems

1. Assume that $f : [a, b] \to \mathbb{R}$ is continuous, and f is differentiable on (a, b). Assume in addition that the function $f'(x) : (a, b) \to \mathbb{R}$ is Darboux integrable.* Show that

$$\int_a^b f'(x)\, dx = f(b) - f(a)$$

Hint: show that $m_j(x_j - x_{j-1}) \leq f(x_j) - f(x_{j-1}) \leq M_j(x_j - x_{j-1})$.

* Define the Darboux integral of $g : (a, b) \to \mathbb{R}$ just as for g on $[a, b]$, but let $M_1 = \sup_{x\in(a,x_1]} g(x)$, $M_n = \sup_{x\in[x_{n-1},b)} g(x)$, in the Darboux integral definition, and similarly for m_1, m_n.

2. (a.) Use the intermediate value and mean value theorems to show that if $y > 0$ is real and $k \geq 1$ is a natural number, then there is a unique $x > 0$ so that $x^k = y$.

(b.) Denote the x in part (a) as $y^{1/k}$. Show that for each $y > 0$ we have $\lim_{k\to\infty} y^{1/k} = 1$. (You may use that $\lim_{k\to\infty} r^k = 0$ if $0 < r < 1$, and $\lim_{k\to\infty} r^k = \infty$ (properly interpreted) if $r > 1$, both of which we proved in 424.