Math 425 Autumn 2018

Homework 3 **Due Monday, January 29**

Chapter 4 [Pugh, pg. 263]: 1(a), 2, 3, 4(a).

Additional problems

1. Define $C_0(\mathbb{R}) \subset C_b(\mathbb{R})$ to be the subspace consisting of continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that $\lim_{x\to\infty} f(x) = 0$, in the sense that

$$\forall \epsilon > 0 \ \exists R \ : \ |f(x)| < \epsilon \ \text{ if } \ |x| > R.$$

(a.) Show that $C_0(\mathbb{R})$ is a closed subset of $\big(C_b(\mathbb{R}), \|\cdot\|_u\big)$.

(b.) Show that if $f \in C_0(\mathbb{R})$ then f is uniformly continuous.

(c.) Find an example of $f \in C_b(\mathbb{R})$ which is not uniformly continous.

2. Suppose that $R > 0$ and $f : (-R, R) \to \mathbb{R}$ has derivatives of all order, and that for some constant $C > 0$

$$\sup_{x\in(-R,R)} |f^{(k)}(x)| \le \frac{C\,k!}{R^k}.$$

Show that the power series $\displaystyle\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!}\, x^k$ converges, and equals $f(x)$, for $|x| < R$.