Math 425 Winter 2018

Homework 4 **Due Monday, February 5**

Chapter 4 [Pugh, pg. 263]: 9, 12, 13, 15, 19.

For Problem 15, show that the result holds for functions $f \in C_b(\mathbb{R})$; that is, bounded continuous functions on the real line. Do this for both part (a) and part (b).

Additional problem

1. Assume that, for some $R > 0$ and some C, $|a_k| \leq C\,R^{-k}$ for all k. Consider the function $f(x) = \sum_{k=0}^{\infty} a_k x^k$ for $x \in (-R, R)$. Show that

$$|f^{(m)}(x)| \leq \frac{C\,m!}{R^m}\left(1 - \frac{|x|}{R}\right)^{-m-1} \quad \text{for } x \in (-R, R).$$

Hint: consider $g(x) = (1 - x/R)^{-1}$ and the series expansion for $g^{(m)}(x)$.