Math 425 Winter 2018
Homework 5 **Due Wednesday, February 21**

Chapter 4 [Pugh, pg. 263]: 26, 27, 34(a,b,c).

Additional problems

1. Suppose that $f \in C^0([a,b])$ satisfies $\int_a^b f(x)q(x)\,dx = 0$ for every polynomial function $q(x)$. Show that $f(x) = 0$ for all $x \in [a,b]$.

2. This problem fills in the details of a result stated in lecture,

(a.) Using the fact that $x \to e^{-x}$ is a homeomorphism of $[0,\infty)$ onto $(0,1]$ (with inverse $\log y$), show that every function $f \in C_b([0,\infty))$ can be written in the form $f(x) = g(e^{-x})$ for some $g \in C_b((0,1])$.

(b.) Show that if $f \in C_b([0,\infty))$ satisfies $\lim_{x\to\infty} f(x) = 0$, then the function $g(x)$ defined above extends to a continuous function on $[0,1]$ by setting $g(0) = 0$.

(c.) Show that if $f \in C_b([0,\infty))$ satisfies $\lim_{x\to\infty} f(x) = 0$, and $\epsilon > 0$, then there exists n and a finite set of numbers $\{a_1, a_2, \ldots, a_n\}$ so that

$$\sup_{0\le x<\infty} \left| f(x) - \sum_{j=1}^{n} a_j e^{-jx} \right| < \epsilon.$$

3. This problem replaces problem 24 from the text.

(a.) Suppose that $f_n \in C^0([a,b])$ are a monotonically decreasing family of nonnegative functions, $f_1(x) \ge f_2(x) \ge \cdots \ge 0$, which converge pointwise to 0 on $[a,b]$. Show that f_n converges uniformly to 0.

Hint: the proof is similar to the proof that pointwise continuity of a function on $[a,b]$ implies uniform continuity, with N instead of δ.

(b.) Give an example of $f_n \in C_b(\mathbb{R})$ that are monotonically decreasing and converge pointwise to 0, but do not converge uniformly to 0.