

Math 427 Final Exam Practice Problems

1. Find the order of the zero of $f(z)$ at z_0 , for:

(a.) $f(z) = z(\sin z)^2$, $z_0 = \pi$.

(b.) $f(z) = (z^2 + 1)^3$, $z_0 = i$.

(c.) $f(z) = (z^2 + 4\pi^2)(e^z - 1)$, $z_0 = 2\pi i$.

2. (a.) Find the radius of convergence of the Taylor expansion of $\frac{e^z}{z^2 + 1}$ about $z_0 = \frac{1}{2}$.

(b.) Find the values of a_0, a_1, a_2 , in the series expansion $\log(1 + \sin z) = \sum_{k=0}^{\infty} a_k z^k$ about $z_0 = 0$.

Here, \log is the principal branch of the logarithm. (You do not need to find the other a_k .)

3. Find the isolated singularities of the following functions, and say whether they are removable singularities, poles, or essential singularities.

If one of the singularities is a pole, find the principal part of the function at one of the poles (you can choose which one).

(a.) $\frac{z^3}{\sin z}$

(b.) $\frac{e^z}{(z^2 + 1)^2}$

(c.) $\frac{e^{2z} - 1}{z}$

(d.) $z^2 \sin\left(\frac{1}{z}\right)$

(e.) $\frac{\cos z}{(z^2 - \pi^2/4)^2}$

4. Evaluate the following contour integral; the contour goes counter-clockwise around $\partial D_2(2)$.

$$\int_{|z-2|=2} \frac{e^z}{(z-1)(z+1)} dz$$