Name: **Student #:**

Math 427 Midterm, Autumn 2016

Answer problems on the pages I handed out.
Staple together the pages you want considered, in order, with this page on top.

1. For each of the following sets, say whether the set is: **open**, **closed**, **convex**. (That is, each item will have 3 yes/no answers). There is no need to justify your answers.

(a.) The set of $z \in \mathbb{C}$ such that $\mathrm{Re}(z) = 0$ and $0 < \mathrm{Im}(z) < 1$.

(b.) The set of $z \in \mathbb{C}$ such that $|z - 1| = 1$.

(c.) The set of z such that $\mathrm{Im}(\mathrm{z}) > \mathrm{Re}(z)^2$.

2. Use the Cauchy–Riemann equations to determine if each of the following functions is analytic. If it is, find $f'(x + iy)$ (you may leave the answer in the $u(x, y) + iv(x, y)$ form).

(a.) $f(x + iy) = x^2 - y^2 + x + i(2xy - y)$

(b.) $f(x + iy) = e^y \cos x - i\, e^y \sin x$

3. Find $\displaystyle\int_\gamma f(z)\, dz$, where $\gamma(t) = 1 + t + it\,,\ t \in [0, 1]\,,$ that is, $\gamma = [1, 2 + i]$, for the following functions. You may use any methods we learned to evaluate path integrals.

(a.) $f(z) = z^2$

(b.) $f(z) = |z|^2$

4.

(a.) Find all complex numbers z such that $z^3 = 1 - i$. Express the answers in the form $x + iy$ (i.e. don't leave them in polar form). The answers for x and y can be given using sin and cos functions.

(b.) Let $0 < r < R$ be real numbers. Describe the set of $z \in \mathbb{C}$ such that $r < |e^z| < R$.

(c.) Describe the image $f(E)$ where E is the set $\{z \in \mathbb{C} : \mathrm{Im}(z) > 0 \text{ and } \mathrm{Re}(z) > 0\}$, and $f(z) = z^3$. As part of your answer, sketch the image in the complex plane. Indicate whether any boundary lines are included or not in the image.

5. Express the following functions in the form $f(x + iy) = u(x, y) + iv(x, y)$ for real-valued functions u and v. That is, give explicit formulas for $u(x, y)$ and $v(x, y)$.

(a.) $f(z) = z \log z\,,$ on the set $\mathrm{Re}(z) > 0$, where $\log z$ is the principal branch.

(b.) $f(z) = \dfrac{e^z}{z}\,,$ on the set $z \neq 0$.