

Name :

Student # :

Math 428 Final Exam, Winter 2019

Answer all problems on supplemental pages.

Staple together the pages you want considered, in order, with this page on top.

1. Show that all of the zeroes of $f(z) = z^9 - 96z^2 + 25$ lie in the set $\{z : \frac{1}{2} < |z| < 2\}$.

2. For each of the following examples, find a 1-1 conformal of E onto F .

(a.) $E = \{z : 0 < \operatorname{Im}(z) < \pi\}, \quad F = \{z : \operatorname{Re}(z) > 0\}.$

(b.) $E = \{z : |z| < 1\}, \quad F = \mathbb{C} \setminus (-\infty, 0].$

3. (a.) Evaluate the following integral using residues: $\int_0^{2\pi} \frac{1}{3 - 2 \cos t} dt.$

(b.) Evaluate the following Fourier transform for $s \neq 0$: $\int_{-\infty}^{\infty} e^{-ist} \frac{t^2}{(t+i)^3} dt.$

4. Find the residue at $z = 0$ for the following functions. You may use your favorite method (but show your work).

(a.) $f(z) = \frac{1}{z^2 \sin z}.$

(b.) $f(z) = \frac{\cos z}{z \cos z - \sin z}.$

5. Evaluate the following integral using residues: $\int_{-\infty}^{\infty} \frac{\sin t}{t(t^2 + 1)} dt.$

6. (a.) Find the set of $z \in \mathbb{C}$ so that $f(z) = e^{z^3 + 6z - 20}$ has a local analytic inverse f^{-1} at z .

(b.) Find $(f^{-1})'(1)$ if f^{-1} is a local inverse for f such that $f^{-1}(1) = 2$.