Math 428 Winter 2020

**Homework 1** **Due Wednesday, January 15**

**Section 3.5**: 7, 11

**Section 4.1**: 2, 3, 13

Additional problems:

**1.** Consider the cycle $\Gamma = \left\{\partial D_3(0)\,,\, -\partial D_{\frac{1}{2}}(-1)\,,\, -\partial D_{\frac{1}{2}}(1)\right\}$. Sketch the cycle, and indicate the values of $\mathrm{ind}_\Gamma(z)$ on each component of $\mathbb{C}\setminus\Gamma$. In particular, show that $\mathrm{ind}_\Gamma(1) = \mathrm{ind}_\Gamma(-1) = 0\,.$

**2.** Suppose that $f$ is analytic on the set $E = \mathbb{C}\setminus\{-1,1\}$. That is, $f$ has isolated singularities at $z = 1$ and at $z = -1$, and is analytic everywhere else. Use the results of the previous problem to show that

$$\int_{\partial D_3(0)} f(w)\,dw \;=\; \int_{\partial D_{\frac{1}{2}}(-1)} f(w)\,dw \;+ \int_{\partial D_{\frac{1}{2}}(1)} f(w)\,dw$$