## Name:

Student #:

## Math 428 Midterm, Winter 2019

- Answer all problems on supplemental pages.
- Staple together the pages you want considered, in order, with this page on top.
- 1. Find all poles of the following functions, find their order, and find the residue of f at each of the poles.

(a.) 
$$f(z) = \frac{z}{e^z - 1}$$
  
(b.)  $f(z) = \frac{z^2}{(z^2 - 4)^2}$ 

**2.** Find the Laurent expansion of  $\frac{\sin z - z}{z^6}$  about z = 0, and use it to find  $\operatorname{Res}\left(\frac{\sin z - z}{z^6}, 0\right)$ .

**3.** Calculate the following contour integrals. In part (b.)  $\log z$  is the principal branch.

(a.) 
$$\int_{|z-i|=1} \frac{e^z}{z^2+1} dz$$
  
(b.)  $\int_{|z-1|=\frac{1}{2}} \frac{e^z}{\log z} dz$ 

- **4.** (a.) Find the maximum modulus of  $f(z) = e^z$  on the set  $|z| \le 2$ .
  - (b.) Show that if c is a complex number with |c| < 3 then  $z^5 ce^z$  has fives zeroes in |z| < 2.