Math 525, Winter 2008, Homework 4

The **Midterm Exam** will be **Wednesday, February 13**.

The following homework is due Wednesday, February 13.

1. Let μ be a finite signed measure on $\mathbb{R}$, and $F(x) = \mu((-\infty, x])$. Show that $|\mu|(\mathbb{R})$ equals the total variation of F on $\mathbb{R}$.

(The assumption is that F takes real values; the complex version of this problem is more involved.)

[Hint: The direction $\geq$ is straightforward. For the converse, apply Theorem 1.20 to $|\mu|$ to find $A =$ a finite union of h-intervals such that $|\mu|(A\Delta P) = |\mu|(A^c\Delta N) < \epsilon$, where $\mathbb{R} = P \cup N$ is a Hahn decomposition for μ. Strictly speaking Theorem 1.20 gives open intervals, but by continuity from inside you can take slightly smaller h-intervals instead.]

Remark: applying this to the restriction of μ to $(-\infty, x]$ shows that $|\mu|((-\infty, x]) = T_F(x)$. This shows that the Jordan decomposition $\mu = \mu^+ - \mu^-$ corresponds to the Jordan decomposition $F = F^+ - F^-$.

2. Problem 39, page 108 of Folland.

3. Problem 40, page 109 of Folland.