MATH/STAT 394 HOMEWORK 3
DUE ON JAN 30, 2019

Problem 1. We roll two dice. Find the conditional probability that at least one of the rolls is an even number, given that the sum is 8.

Problem 2. We have two urns. The first urn contains two balls labeled 1 and 2. The second urn contains three balls labeled 3, 4, and 5. We choose one of the two urns at random with equal probability and then sample one ball, uniformly at random, from the chosen urn. What is the probability that we picked the ball labeled 5?

Problem 3. We have two urns. The first urn contains three balls labeled 1, 2, and 3. the second urn contains four balls labeled $2, 3, 4$ and 5. We choose the first urn with probability $1/5$ and the second urn with probability $4/5$. then we sample, uniformly at random, one ball from the chosen urn.

(i) What is the probability we pick the ball labeled 2?
(ii) Suppose that ball 3 was chosen. What is the probability that it came from the second urn?

Problem 4.

(i) Use the definition of conditional probability to show that $P(A^c \mid B) = 1 - P(A \mid B)$.
(ii) Suppose that $P(A \mid B) = 0.6$ and $P(B) = 0.5$. Find $P(A^c \cap B)$.

Problem 5. An urn contains 11 balls, 3 white, 3 red, and 5 blue balls. Take out two balls at random, without replacement. You win \$1 for each red ball you select and lose a \$1 for each white ball you select. Let X be the random variable that notes the amount you win. Find the probability mass function (pmf) of X.

Problem 6. The Acme Insurance company has two types of customers, careful and reckless. A careful customer has an accident during the year with probability 0.01. A reckless customer has an accident during the year with probability 0.04. 80% of the customers are careful and 20% of the customers are reckless. Suppose a randomly chosen customer has an accident this year. What is the probability that this customer is a careful customer?

Problem 7. We choose a number from the set $\{1, 2, 3, \ldots, 100\}$, uniformly at random. Let X be the chosen number. For each of the following choices determine if the two events are independent or not.

(i) $A = \{X \text{ is even}\}$, $B = \{X \text{ is divisible by } 5\}$.
(ii) $C = \{X \text{ has two digits}\}$, $D = \{X \text{ is divisible by } 5\}$.
(iii) $E = \{X \text{ is a prime number}\}$, $F = \{X \text{ has a digit } 5\}$.

Problem 8. Suppose that $P(A) = 1/3$, $P(B) = 1/3$, and $P(A \cap B^c) = 2/9$. Are A and B independent? Why or why not?