Homework #1 Due Wednesday October 8

1. Chapter 1 # 2.4
2. Chapter 1 # 3.14
3. Appendix A # 3.1
4. $\{0,1\}^{\mathbb{N}}$ is the space of infinite sequences $(x_1, x_2, \dots)$ of zeros and ones. For any finite (possibly empty) sequence s of zero and ones, define $A_s \subset \{0,1\}^{\mathbb{N}}$ to be the set of all infinite sequences that extend s. Let $\mathcal{F}$ be the class of finite unions of sets $\{A_s\}$ and $\mathcal{B} = \sigma\{A_s : s \in \text{ Finite sequences}\}$.
   (a) If B_n is a sequence of disjoint sets in $\mathcal{F}$ and $\cup B_n = \{0,1\}^{\mathbb{N}}$, then show that $B_n = \emptyset$ after some stage.
   (b) Show that any finitely additive probability on $\mathcal{F}$ is countably additive.
   (c) Show that there is a unique probability measure λ on $\mathcal{B}$ such that $\forall s \in$ Finite sequences,
   $$\lambda(A_s) = \frac{1}{2^{\text{length of } s}}.$$
5. Let X be a $\mathbb{R}$-valued random variable such that $0 \le \mathbb{E}(X)$ and $0 < \mathbb{E}(X^2) < \infty$, and let $\lambda \in [0,1]$. Then
   $$\mathbb{P}\Big(X > \lambda \mathbb{E}(X)\Big) \ge (1-\lambda)^2 \frac{(\mathbb{E}X)^2}{\mathbb{E}(X^2)}.$$