

RESEARCH STATEMENT

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My current research focuses are specified in the following three aspects: inverse problems for PDEs, in particular inverse scattering and EIT&Calderón problem with partial data; designs and analyses of invisibility and cloaking devices; and applied and numerical analysis, in particular asymptotic analysis in scattering theory and their applications, qualitative numerical reconstruction algorithms for inverse problems and geometric numerical integration for dynamical systems.

1. INVERSE PROBLEMS FOR PARTIAL DIFFERENTIAL EQUATIONS

1.1. Inverse acoustic and electromagnetic scattering problems. Roughly speaking, the scattering problems are concerned with the effect an inhomogeneity (*scatterer*) has on an incident (acoustic or electromagnetic) wave. The *forward scattering problem* is to study the (far-field or near-field) behavior of the scattered wave. Of possibly even more interest is the *inverse scattering problem* of extracting physical nature of the scatterer from a knowledge of the (far-field or near-field) behavior of the scattered wave field; i.e., to reconstruct the differential equation and/or its definition domain from certain measurement of its solution(s). Here the governing PDE is scalar wave equation for acoustic scattering and Maxwell's equations for electromagnetic scattering. Such inverse problem has been playing an indispensable role in real life, and forms the basis of many areas of science and engineering, such as radar, sonar, geophysical exploration, non-destructive testing, and medical imaging, etc. (see, e.g., [6]). In the following, we take the inverse acoustic scattering as an example for illustration. Consider the Helmholtz system

given as follows,

$$(1.1) \quad \begin{cases} \Delta u + k^2 m(x)u = 0 & \text{in } \mathbb{R}^n \setminus \bar{D}, \\ u = u^i + u^s & \text{in } \mathbb{R}^n, \\ \mathcal{B}u = 0 & \text{on } \partial D, \\ \mathcal{M}u = 0, \end{cases}$$

where

- a) $D \subset \mathbb{R}^n$ is a bounded Lipschitz domain with connected complement, which denotes an impenetrable obstacle.
- b) $m(x) \in L^\infty(\mathbb{R}^n \setminus \bar{D})$ with $\text{supp}(1 - m) \cup \bar{D} := \Omega \Subset \mathbb{R}^n$, which represents the *refractive index* of an penetrable acoustic medium supported in $\Omega \setminus \bar{D}$.
- c) u^i is an entire solution to the PDO $(\Delta + k^2)$, denoting the incident field. Usually, we take u^i to be the plane wave $\exp\{ikx \cdot d\}$, where k is the wave number and $d \in \mathbb{S}^{n-1}$ is the incident direction. u^s is known as the scattered wave.
- d) $\mathcal{B}u = 0$ is the boundary condition, which might be the Dirichlet BC corresponding to a *sound-soft* obstacle, or Neumann boundary BC corresponding to a *sound-hard* obstacle, or Robin BC corresponding to an *impedance-type* obstacle, or of much more complicated nature of mixed BCs.
- e) $\mathcal{M}u = 0$ is the so-called radiation condition, e.g., if we take $u^i = \exp\{ikx \cdot d\}$, this is the so-called Sommerfeld radiation condition given by

$$\lim_{|x| \rightarrow \infty} |x|^{(n-1)/2} \left(\frac{\partial u^s}{\partial |x|} - iku^s \right) = 0.$$

We remark that it may happen that $D = \emptyset$, and this corresponds to the case that the scatterer consists solely of medium, and it may also happen that $m = 1$ in $\mathbb{R}^n \setminus \bar{D}$ and this corresponds to the case that the scatterer consists solely of obstacle. u^s admits asymptotically as $|x| \rightarrow \infty$ the following development

$$(1.2) \quad u^s = \frac{e^{ik|x|}}{|x|^{(n-1)/2}} \left\{ A(\hat{x}; k, d) + \mathcal{O}\left(\frac{1}{|x|}\right) \right\},$$

where $\hat{x} = x/|x| \in \mathbb{S}^{n-1}$ denoting the observation direction. The analytic function $A(\hat{x}; k, d)$ is known as the *scattering amplitude* or *far-field pattern*. Now, the inverse problem is that

Can one recover D and/or $m(x)$ from $A(\hat{x}; k, d)$?

There is much progress in this inverse problem, but more remains challengingly unsolved (see [6], [23] and [48]). One of the key issues is the *uniqueness/identifiability*, i.e., *is the correspondence between $A(\hat{x}; k, d)$ and $D/m(x)$ one to one?* The uniqueness is of significant importance in that it ensures that we can really identify the target object with measurement and it also provides how much measurement data one should use for the identification. For a scattering obstacle located in homogeneous background medium (where $m(x) = 1$), there is a widespread belief that one can establish the uniqueness with a single far-field measurement, namely, $A(\hat{x}; k, d)$ with k, d fixed. However, this well-known problem remains largely open with only partial progress on obstacles with restrictive physical and geometric conditions (see, e.g. [4]). Here, we cite a remark from [23] which is read as: *This is a well-known question that supposedly can be solved by elementary means. However, it has been open for thirty to forty years.* In the past few years, significant progress has been

achieved for the unique determination of general polyhedral scatterers under various realistic settings with optimal far-field measurements. This is mainly based on the reflection principles for solutions of Helmholtz equation, and more crucially, the *path argument* originally developed in Liu-Zou [37]. In [33] and [34] by Liu-Yamamoto-Zou, completely novel reflection principles for Maxwell's equations are established for the first time, and based on which all the above mentioned uniqueness have been extended to the electromagnetic scattering problems. We refer to [36] by Liu-Zou for a comprehensive review in this aspect and related literature. A recent effort is to study the unique determination of obstacles with piecewise analytic boundaries. The main idea is to study the microlocal analytic singularities of the scattering wave fields near the boundary of the obstacle and thus identifying it. Another project is to establish uniqueness of determining moving obstacles with dynamical measurement data (see, e.g. [45]).

The previously mentioned study is basically conducted in the setting with homogeneous background space. It is natural to consider the unique determination of obstacles buried in certain inhomogeneous medium. That is, *can one recover $m(x)$ and (possibly buried) D from $A(\hat{x}; k, d)$* ? If one takes the measurement data to be $A(\hat{x}; k, d)$ for all k and $\hat{x}, d \in \mathbb{S}^{n-1}$, some results can be found in literature (see, e.g., [8]) on reconstructing $m(x)$ with known restricted D . However, it is natural to take k fixed whereas $\hat{x}, d \in \mathbb{S}^{n-1}$ and this is known as inverse scattering problem at a fixed frequency. If $D = \emptyset$, namely, there is no obstacle present, the uniqueness is established in [46] where the famous Sylvester-Uhlmann's method of constructing CGO solutions is introduced and this is also connected to the EIT&Calderón problem in the subsequent section. Whereas if $m(x)$ is known a priori, unique determination of D is established in [25] and [39]. In Liu [31], the uniqueness of simultaneously reconstructing the buried obstacle D and surrounding medium $m(x)$ is established for the first time, but under the assumption that the buried obstacle D is of polyhedral type. A recent effort is to extend this uniqueness result to more general setting and to inverse electromagnetic scattering problems.

1.2. EIT and Calderón problem with partial data. The Calderón problem is whether one can determine the conductivity of the subsurface of the Earth by making voltage and current measurements at the surface. The problem of determining the electrical properties of medium by making voltage and current measurement at the boundary has also raised the interest of the medical imaging community and known as Electrical Impedance Tomography (EIT).

Let $\Omega \subset \mathbb{R}^n, n \geq 2$, be a bounded Lipschitz domain. The electrical conductivity of Ω is represented by a bounded and positive function $\gamma(x)$. In the absence of sinks or sources of current the potential $u \in H^1(\Omega)$ with given boundary voltage potential $f \in H^{1/2}(\partial\Omega)$ is a solution of the Dirichlet problem

$$(1.3) \quad \operatorname{div}(\gamma \nabla u) = 0 \quad \text{in } \Omega, \quad u|_{\partial\Omega} = f.$$

The Dirichlet to Neumann (DN) map, or voltage to current map, is given by

$$\Lambda_\gamma(f) = \left(\gamma \frac{\partial u}{\partial \nu} \right) \Big|_{\partial\Omega},$$

where ν denotes the unit outer normal to $\partial\Omega$. The inverse problem is to determine γ knowing Λ_γ . As in the inverse scattering problems, the first issue we shall consider in this context is the uniqueness/identifiability, i.e., *can Λ_γ determine γ* ? Substantial

progress has been made since Calderón’s pioneering work. A crucial ingredient in Calderón’s approach is the use of the harmonic complex exponential solutions:

$$u = e^{x \cdot \rho}, \text{ where } \rho \in \mathbb{C}^n \text{ with } \rho \cdot \rho = 0.$$

In [46, 47], Sylvester and Uhlmann constructed in dimension $n \geq 2$ complex geometrical optics (CGO) solutions to the conductivity equation of the following form

$$u = e^{x \cdot \rho}(1 + \psi(x, \rho)),$$

where $\psi(x, \rho)$ is a remainder term with ‘small’ norm. The construction of the CGO solutions and the study of the uniqueness issue can be reduced to study the Schrödinger equation with potential $q = \Delta\sqrt{\lambda}/\sqrt{\gamma}$. These solutions were used in [46] to show in dimension $n \geq 3$ that Λ_γ determines uniquely γ . In dimension 2, the construction of CGO solutions combined with $\bar{\partial}$ method can be used to show similar uniqueness (see [40]). We refer to [3, 49, 50] for a state-of-art survey and related literature. It is interesting to mention that the Calderón problem is of geometric nature and related to the boundary rigidity problem in geometric analysis of determining a Riemannian manifold from boundary distance function.

In the past few years, much attention has been paid on how to establish the unique identifiability with only partial boundary measurement, and this is also known as *local problem*. Let $\Gamma, \Gamma' \subset \partial\Omega$ be two open patches. Let the voltage input be as $f \in H^{1/2}(\partial\Omega)$ with $\text{supp } f \subset \Gamma$. The inverse problem is that can one determine γ from $\Lambda_\gamma(f)|_{\Gamma'}$. The recent significant progress is that in [24], for dimension $n \geq 3$, the unique identifiability is proved when Γ' is bigger than the complement of Γ . Whereas in dimension 2, the uniqueness is established in [41] when $\Gamma = \Gamma'$. These results are based on constructing more general CGO solutions of the following form

$$u = e^{\tau\phi}(a + r),$$

where ϕ is a phase function and r is a remainder term with ‘small’ norm. Our recent study is to establish more general local uniqueness result but restrict to the case that Ω is a polyhedral domain, where one can make of the reflection principles to Schrödinger equation to construct CGO solutions with vanishing boundary data.

2. INVISIBILITY AND CLOAKING

The goal of cloaking is to coat an object with artificially designed material so that one can steer waves/lights around the object. In doing this, one can achieve customized effects on wave propagation and under wave detection, the object can be virtually reshaped or even invisible. Several different approaches have emerged, including one based on “anomalous localized resonance” and another based on “transformation optics”. Recently, there is an avalanche of study on designs of various striking cloaking devices; e.g., invisibility cloaking devices [14, 22]; field rotators [2]; concentrators [28]; electromagnetic wormholes [9, 10], superscatters [51] etc.. We refer to the most recent survey papers [11] and [12] for a comprehensive review and related literature. The crucial observation is that certain PDEs governing the wave phenomena are form-invariant under transformations, e.g., Helmholtz equation for acoustic scattering and Maxwell’s equations for electromagnetic scattering. Hence, one could form new acoustic or EM material parameters (in the physical space) by pushing forward old ones (in the virtual space) via a mapping F . Such materials/media are called *transformation media* [42], which are among the metamaterials or structured media. It turns out that the wave solutions in

the virtual space with the old material parameters and in the physical space with the new material parameters are also related by the push-forward F . Those key ingredients pave the way for the design of optical devices with customized effects on wave propagation.

Let g be a Riemannian metric. The wave scattering associated with the metric g is governed by the reduced wave equation

$$(\Delta_g + k^2)u = 0,$$

where the Laplace-Beltrami operator associated with g is given in local coordinates by

$$\Delta_g u = \frac{1}{\sqrt{|g|}} \sum_{i,j=1}^3 \frac{\partial}{\partial x_i} \left(\sqrt{|g|} g^{ij} \frac{\partial u}{\partial x_j} \right).$$

In acoustic scattering, $\sigma = (\sigma^{ij})_{i,j=1}^3$ with $\sigma^{ij} := \sqrt{|g|} g^{ij}$ is the anisotropic acoustic density and $\sqrt{|g|} = |\sigma|$ is the bulk modulus, where $(g^{ij})_{i,j=1}^3$ is the matrix inverse of the matrix $(g_{ij})_{i,j=1}^3$, and $|g| = \det g$, $|\sigma| = \det \sigma$. For a smooth diffeomorphism $F := \mathcal{M}_1 \rightarrow \mathcal{M}_2$, $y = F(x)$, the metric $g(x)$ transforms as a covariant symmetric 2-tensor,

$$(2.1) \quad \tilde{g}_{ij}(y) := (F_*g)_{ij}(y) = \sum_{l,m=1}^3 \frac{\partial x^l}{\partial y^i} \frac{\partial x^m}{\partial y^j} g_{lm} \Big|_{x=F^{-1}(y)},$$

and then, for $u = \tilde{u} \circ F$, we have

$$(2.2) \quad (\Delta_g + k^2)u = 0 \iff (\Delta_{\tilde{g}} + k^2)\tilde{u} = 0.$$

Similarly, the Maxwell's equations can also be formulated in local coordinates as follows,

$$(2.3) \quad dE = ik *_g H, \quad dH = -ik *_g E,$$

where $E = E_j dx_j$ and $H = H_j dx_j$ are differential 1-forms, and $*_g$ denote the Hodge-operator on 1-forms given by

$$*_g(E_j dx^j) = \frac{1}{2} |g|^{1/2} g^{jl} E_j s_{lpq} dx^p \wedge dx^q = \frac{1}{2} \varepsilon^{jl} E_j s_{lpq} dx^p \wedge dx^q,$$

with s_{lpq} denoting the Levi-Civita permutation symbol, and $s_{lpq} = 1$ (resp. $s_{lpq} = -1$) if (l, p, q) is an even (resp. odd) permutation of $(1, 2, 3)$ and zero otherwise. Clearly, we also have that the Maxwell's equations are transformation invariant. Those transformation invariance facts forms the basics for transformation-media based cloaking devices.

In [15, 26, 42], radial transformation is introduced which blows up a single point to a metric ball. Invisibility cloaking medium are derived based on such transformation. Clearly, the transformation medium in this way is intrinsically singular. Correspondingly, the transformed PDEs in the physical space become singular. Therefore, in order to rigorously justify the perfect cloaking, we need to deal with the singular PDEs. For perfect cloaking of conductivity equation, which can be considered as optics at zero frequency, the invisibility is mathematically justified in [16] by using the removability of point singularities for harmonic functions; whereas an alternative treating is provided in [22], where *near-invisibility* is introduced from a regularization viewpoint and the invisibility is rigorously justified based on certain stability estimates for conductivity equation with small inclusions. For the

finite frequency cases, a novel notion of *finite energy solutions* is introduced in [14] and the invisibility cloaking of acoustic and electromagnetic medium are then justified directly. In Liu [32], we proved the near-invisibility cloaking for both acoustic and electromagnetic obstacles. To our knowledge, this is the first result on near-invisibility cloaking for wave scattering at positive frequency. However, all the approximate cloakings are conducted with the background space being homogeneous. A recent effort is to extend those results to the more general and realistic setting that the background space is inhomogeneous.

As we mentioned earlier, there is another route of treating the singular PDEs underlying the invisibility cloaking in [14]. However, in [14], the cloaked region is basically a metric ball, where the local coordinate decomposition is applied in pertinent arguments. Our another recent contribution along this line is to construct and investigate much more general active cloaking devices (see Hetmaniuk-LeVeque-Liu-Uhlmann [18]). To deal with the singular PDEs, we introduce the weighted Sobolev spaces with degenerate weights, encompassing and generalizing the idea of finite energy solutions in [14]. We also establish some crucial properties on these weighted Sobolev spaces and rigorously justify those cloaking devices. Motivated by the properties of the finite energy solution near the cloaking interface, we present a completely novel finite elements discretization to solve the governing singular Helmholtz equation. The numerical experiments illustrate the performance of this discretization and highlight that a naive use of classical finite elements leads to inaccurate solutions. In [19], we also consider the finite energy solutions to two dimensional cloaking devices for the first time. A new hidden boundary condition is discovered at the exterior cloaking interface. Novel finite elements discretizations are also developed for the two dimensional cloaking problem. Our recent effort is to generalize those results to electromagnetic scattering.

As we mentioned earlier, the cloaking medium are inevitably singular and anisotropic, which pose much challenges for building those materials. Recently, the trend is to derive nonsingular and isotropic cloaking materials with only approximate invisibility effect. In [13], approximate nonsingular isotropic cloaking is derived for quantum and acoustic scattering. The basic idea is to first approximate the singular cloaking by nonsingular but still anisotropic cloaking, and then to derive approximate nonsingular isotropic cloaking by inverse homogenization theory. Our project along this line is to study the approximate isotropic cloaking for obstacle scattering and electromagnetic scattering.

In addition to the cloaking with transformation medium, there is another novel route to attain invisibility by introducing an active source which cancel the scattering wave fields due to the target scatterer (see [38]). However, the invisibility is only justified for optics at zero frequency. A joint effort with D. Onofrei is to construct the exterior cloaking devices at finite frequency. There, the problem can be reformulated as a control problem in finding a suitable active source emitting waves to cancel the scattering wave from the target scatterer.

3. APPLIED AND NUMERICAL ANALYSIS

3.1. Acoustic and electromagnetic scattering theory and numerics. Scattering theory has played a central role in twentieth century mathematical physics. In fact, as Whitham put it, “almost any field of science and engineering involves

some questions of wave motion”. I particularly interested in studying the asymptotical behaviors of wave phenomena in extreme settings, namely, the classical conditions on the governing PDEs are violated, e.g., the coefficients of the PDEs become singular, the distance of two scatterers in the multiple scattering becomes too close or too far away etc. Those studies may find applications in various problems, e.g., in Li-Liu-Zou [30], the asymptotic behavior of scattering wave field with respect to the size of the underlying scatterer, or distance between two scatterers from a multiple scattering system could be used to derive some important properties of linear sampling method. A recent project is to establish the connections between scattering obstacles and scattering media. Basically, we would show that the obstacles are exactly the extreme limits of medium when the material parameters become singular. Such study would be ended by providing important mathematical techniques in proving unique determination of objects buried in certain medium, and also in numerically reconstructing them. Our another interesting project is to study the scattering due to negative indexed medium (NIM) (see, e.g., [27]), where the material parameter tensors have both positive and negative eigenvalues, and they pose much challenges to mathematics community.

3.2. Qualitative numerical reconstruction algorithms for inverse problems. Inverse scattering problems are known to be nonlinear and ill-conditioned, which makes the qualitative numerical reconstructions much more difficult and challenging. So far, various numerical methods have been developed for reconstructing the scatterer, but as pointed out by Hooper and Hambric in [20] that “*Target identification is the great unsolved problem. We detect almost everything, we identify nothing*”. As a ready example, it is seen that radar has not yet realized its full potential. In recent years, some good progress has been achieved in developing qualitative methods for inverse scattering problems, such as the linear sampling method, factorization method, and the probe method etc. We refer to the recent survey papers [7], [44] and two monographs [1],[43] on this topic. Our recent contributions in [29] and [30] by Li-Liu-Zou further consolidate the linear sampling method and provide much insights into this method. For future research, we would like to include microlocal analysis into the development of novel numerical reconstruction algorithms to attain more qualitative image reconstruction.

3.3. Geometric numerical integration for dynamical systems. A wide class of problems in the study of dynamical systems, which are fundamental to understanding classical mechanics, are classified as Hamiltonian systems. The dynamics modeled by these problems are known to be conservative. In addition, the types of applications for these systems are numerous and cover many fields, including applications in celestial mechanics, particle dynamics, plasma physics, optics and wave motion. Needless to say, the numerical approximations to these dynamical systems are of significant practical importance. Recent trends in the numerical computation of solutions for dynamical systems have changed, and this change can be mainly attributed to seeing a numerical method as a (discrete) dynamical system. The traditional way to discern the quality of a numerical method is to analyze particular trajectories. However, a more favorable viewpoint is to compare the dynamical behavior of a numerical scheme to that of the original dynamical system for which it is being used to solve. This new viewpoint has led to a field of study all of its own, known as geometric integration (see [17]). The main idea behind geometric

integration is to preserve the underlying structure or certain geometric properties (in phase space) of particular equations. For example, there are methods that give numerical flows which lie on a manifold defined by the problem. These include the well-known symplectic methods, or more general Lie group methods. Numerous experiments and theoretical justifications have shown that structure preserving numerical schemes not only improve the qualitative behaviors of numerical solutions but also yield more accurate long-time integrations. Recently, significant progress has been made on the extension of symplectic integrations of Hamiltonian ODEs to Hamiltonian PDEs, which is known as the multi-symplectic integrations (see, e.g. Hong-Liu-Sun [21] and Liu-Zhang [35]). The multi-symplectic integrators preserve the symplectic structures in both spatial and temporal directions, and yield significantly improved qualitative numerical behaviors. As we mentioned earlier, a significant way to justify the geometric numerical integrators is to study the dynamical behavior of the corresponding numerical flows. This is provided by the so called *backward analysis*, where one determines a modified Hamiltonian system whose flow is exactly the same as the numerical flow produced by the geometric integrator. Hence, in order to study the approximation of the numerical integrator, one only needs to compare the dynamical behavior of the modified Hamiltonian system and the original Hamiltonian system. The backward analysis has been systematically and well developed for symplectic integrators but much less developed for multi-symplectic integrators. For our future research, we will make more insightful investigation on the backward analysis to multi-symplectic integrators.

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