Eulerian Gaussian Beams for High Frequency Wave Propagation

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Gaussian beams provide an efficient tool for seismic wave modeling and imaging in the high frequency regime in that Gaussian beams can take into account caustics naturally. The traditional Gaussian beam summation method is based on Lagrangian ray tracing and local ray centered coordinates. Recent advances in Eulerian geometrical optics motivated us to design Eulerian Gaussian beam summation methods for solving Helmholtz equations. Such an Eulerian method was first proposed by Leung, Qian and Burridge (Geophysics, 72(2007)). This Eulerian formulation of Gaussian beam theory adopts global Cartesian coordinates, level sets and Liouville equations, yielding uniformly distributed Eulerian traveltimes and amplitudes in phase space simultaneously for multiple sources. The time harmonic wavefield can be constructed by summing up Gaussian beams with ingredients provided by the Eulerian formulation. There are three advantages of the Eulerian method. The first advantage is that we have uniform resolution of ray distribution. The second advantage is that we can obtain wavefields excited at different sources by varying only source locations in the summation formula. The third advantage is that we can obtain wavefields excited at different frequencies by varying only frequencies in the summation formula. This lecture will first introduce Eulerian Gaussian beams for computing semi-classical solutions of Schrödinger equations (Leung and Qian, Journal of Computational Physics (2009)) and then construct Eulerian Gaussian beams for Helmholtz equations.