Curvelets and the downward continuation and reverse time migration approaches to seismic inverse scattering

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This series of lectures will give an introduction to the mathematical framework for “wave-equation” imaging and seismic inverse scattering. We begin with (I) analyzing the single scattering operator modelling seismic reflection data, the underlying extension operators, and (“true-amplitude”) directional wavefield decomposition leading to the double-square-root (DSR) equation. We then focus on (II) inverse scattering in the downward-continuation approach making use of the DSR propagator, while introducing the depth-to-time conversion operator and the associated normal operator. We discuss and characterize the operator matrix representations with respect to the frame of curvelets. In part III, we focus on inverse scattering in the reverse-time migration approach. We discuss an equivalent Helmholtz equation formulation as well as the possibility of sparsifying the acquisition making use of the fact that curvelets compress the relevant operator kernels. In part IV, we address illumination analysis, accounting for limited acquisition aperture, and microdiffraction theory, as well as retrofocussing and inversion via diagonal matrix approximation and partial reconstruction. In part V, we characterize the range of the single scattering operator via the introduction of annihilators. We introduce curvilinear coordinates through a Riemannian metric, and the curvilinear DSR condition. We discuss and analyze the “wave-equation” angle transform – both in the downward-continuation and reverse-time/Helmholtz-equation approaches – the underlying geometry of broken geodesics, and associated matrix representation which also provides a multi-scale formulation. We base this inverse scattering program on the scalar wave equation, but will comment on the generalization of the reverse-time approach to hyperbolic systems of principal type describing elastic waves and the notion of polarization separation. We cover joint work with Stolk, as well as with Smith and Uhlmann.

This mini-course will be accompanied by a computer lab involving synthetic seismic data and the curvelet transform.