1. Using theorem 1, $p(t) = -tan(t)$, which is continuous everywhere except for $t = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, ...$ Also, $g(t) = sin(t)$, which is continuous everywhere. So the only interval containing t_0 over which p and g are continuous is the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$. So the theorem guarantees a solution here and nowhere else.

(Note that t_0 is implicitly given as 0 by the problem statement.)

2. Using theorem 2, $f(t, y) = tan(y)t + sin(t)$, and $\frac{\partial f}{\partial y} = sec^2(y)t$, which are both continuous except where $y = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, ...$ So for $y_0 \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, ...$ the theorem guarantees a solution defined on some interval $(-h, h)$.

(Note that the discontinuity does not depend upon t, so our solution would be similar if t_0 was equal to some value other than 0.)

(Note also that when $t = 0$, both $f(t, y)$ and $\frac{\partial f}{\partial y}$ could be defined as $= 0$ since they have a t out in front multiplying everything. So one may be tempted to say that the theorem guarantees a unique solution exists even if say $y_0 = \frac{\pi}{2}$. This is not true though since the point $(0, \frac{\pi}{2})$ is a point of discontinuity in the $t - y$ plane. You can see this since you cannot draw a box around that point on which the functions f and $\frac{\partial f}{\partial y}$ are continuous.)