Math 307 Practice Final, Fall 2006

Updates:
Wed 2:32pm. Correction made to Theorem 2
Wed 3:50pm. Sentence added to the end of question 3
Wed 5:30pm. Correction made to Theorem 6

You may make use of the following:

- I will include the **Laplace Transform table** from page 319. I may not include some transforms, and ask you to derive these yourselves.
- **Integrating Factors** Given the following equation

$$y' + p(t)y = g(t),$$

the appropriate integrating factor is: $\mu(t) = e^{\int p(t)\,dt}$.

- **Euler's Method** Given the following initial value problem

$$y' = f(t, y)$$
$$y(t_0) = y_0$$

Euler's method approximates a solution using:

$$y(t + h) \cong y(t) + hf(t, y)$$

Theorem 1. *Suppose that f and f' are continuous on $[0, \infty)$. Suppose further that there exist constants K and a such that $|f(t)| \leq Ke^{at}$ for $t \geq 0$. Then $\mathcal{L}\{f'(t)\}$ exists for $s > a$, and moreover*

$$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$$

Theorem 2. *Suppose that $f, f', \ldots f^n$ are continuous on $[0, \infty)$. Suppose further that there exist constants K and a such that $|f(t)| \leq Ke^{at}$, $|f'(t)| \leq Ke^{at}, \ldots |f^{(n-1)}(t)| \leq Ke^{at}$ for $t \geq 0$. Then $\mathcal{L}\{f^n(t)\}$ exists for $s > a$, and moreover*

$$\mathcal{L}\{f^n(t)\} = s^n\mathcal{L}\{f(t)\} - s^{(n-1)}f(0) - \ldots sf^{(n-2)}(0) - f^{(n-1)}(0)$$

Theorem 3. *If $F(s) = \mathcal{L}\{f(t)\}$ exists for $s > a \geq 0$, and if c is a positive constant, then*

$$\mathcal{L}\{u_c(t)f(t - c)\} = e^{-cs}F(s), \quad s > a$$

Theorem 4. *If $F(s) = \mathcal{L}\{f(t)\}$ exists for $s > a \geq 0$, and if c is a constant,then*

$$\mathcal{L}\{e^{ct}f(t)\} = F(s - c), \quad s > a + c$$

Theorem 5. *If $F(s) = \mathcal{L}\{f(t)\}$, exists for $s > a$, then*

$$F^{(n)}(s) = \mathcal{L}\{(-t)^n f(t)\}$$

exists and is defined for $s > a$

Theorem 6. *Suppose $F(s) = \mathcal{L}\{f(t)\}$ exists for $s > c$, and $a > 0, b \neq 0$. Then*

$$\mathcal{L}^{-1}\{F(as + b)\} = \frac{1}{a}e^{-bt/a} f\left(\frac{t}{a}\right)$$

and the transform $F(as + b)$ exists for $as + b > c$.

1. (10 points) I will as you to prove one of the theorems appearing on the coversheet, or the corresponding theorem about the inverse transform. So you need to know the relation between the forward and inverse Laplace transforms.

2. (10 points) I will ask you to answer one question applying one or more of the theorems from the coverpage. An example of this would be question 20 from chapter 6.3.

3. (50 points) I will ask you to solve 5 initial value problems. The problems will be (approximately) evenly divided among Laplace transform problems, 1^{st} order equations (chapter 2), and 2^{nd} order equations (chapter 3). The format will be identical to the format used on Midterm 1 and Midterm 2. To study for this I suggest doing old exam and practice exams, and homework problems. I will specifically ask you to use certain methods on certain problems. So you can't just rely on always using one method.

4. (10 points) There will be one question on Euler's Method.
   For example: Using Euler's method with stepsize $= \frac{1}{2}$, find an approximation to $y(2)$ if

$$y' + sin(\pi t)y = 0$$
$$y(\frac{1}{2}) = 3$$

Note that there will be no "story" problems or Fourier series problems