Math 403B: Introduction to Modern Algebra, Winter Quarter 2018
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Homework 1
Due: Friday, January 12

Problem 1.1. Goodman 10.1.1

Problem 1.2. Goodman 10.1.2

Problem 1.3. Goodman 10.1.3

Problem 1.4. Goodman 10.1.4

Problem 1.5. Let D_n be the dihedral group which is the symmetry group of the regular n-gon. Recall that D_n has the presentation

$$D_n = \{e, r, r^2, \dots, r^n, j, rj, \dots, r^{n-1}j\}$$

where r is rotation by $\frac{2\pi}{n}$ radians counterclockwise and j is the reflection across the x-axis. Recall also that $jr = r^{n-1}j$.

(a) If $n \geq 3$, show that D_n is solvable but not simple.
(b) Construct three different composition series for D_{12} and for each one, indicate the composition factors. (By the Jordan–Hölder theorem, each composition series should have the same composition factors after reordering.)

Problem 1.6. Goodman 10.3.1

Problem 1.7. Goodman 10.3.4

Problem 1.8. Goodman 10.3.6

Problem 1.9. Let p be an odd prime. Recall that

$$\mathrm{GL}_2(\mathbb{Z}/p) = \{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z}/p \text{ and } \det A \neq 0\}.$$

Let $\mathrm{SL}_2(\mathbb{Z}/p) \subset \mathrm{GL}_2(\mathbb{Z}/p)$ be the subgroup consisting of 2×2 matrices A with $\det A = 1$. Compute the center Z of $\mathrm{SL}_2(\mathbb{Z}/p)$.

Problem 1.10.

(a) Compute the order of $\mathrm{GL}_2(\mathbb{Z}/p)$.
(b) Compute the order of $\mathrm{SL}_2(\mathbb{Z}/p)$.
(c) Let $\mathrm{PSL}_2(\mathbb{Z}/p) = \mathrm{SL}_2(\mathbb{Z}/p)/Z$ where $Z \subset \mathrm{SL}_2(\mathbb{Z}/p)$ is the center. Compute the order of $\mathrm{PSL}_2(\mathbb{Z}/p)$.