# Midterm Examination 

Complex Analysis (Math 427)
Instructor: Jarod Alper
Fall 2018
November 2, 2018
Name:

Read all of the following information before starting the exam:

- You may refer to your hand-written note sheet ( 8.5 "x11", one-sided).
- You may not consult any other sources (calculator, phone, computer, textbook, other students, ...) to assist in answering the exam problems. All of the work will be your own!
- Write clearly!! You need to write your solutions carefully and clearly in order to convince me that your solution is correct. Partial credit will be awarded.
- Good luck!

| Problem | Points |  |
| :---: | :---: | :---: |
| 1 | $(30$ points $)$ | - |
| 2 | $(30$ points $)$ | - |
| 3 | $(40$ points $)$ | - |
| Total | $(100$ points $)$ |  |

Problem 1 (30 points).
(a) For a positive integer $n$, define the complex number $z_{n}=1 /\left(n^{2}+2018 i\right)$. Does the limit $\lim _{n \rightarrow \infty} z_{n}$ exist? If so, what is the limit?
(b) For which values of $z$ does the series $\sum_{n=0}^{\infty} 1 /(n+z)^{2}$ converge?

Problem 2 (30 points). Let $f(z)=1 / z$.
(a) Use the definition of the derivative to show that $f(z)$ is analytic on $\mathbb{C} \backslash 0$.
(b) Verify the Cauchy-Riemann equations for $f(z)$ on $\mathbb{C} \backslash 0$.

Problem 3 (40 points).
(a) Give a counterclockwise parameterization $\gamma$ of a circle of radius 1 centered at $1+i$.
(b) Compute

$$
\int_{\gamma} \bar{z} d z
$$

(c) Compute the integral

$$
\int_{\gamma} \frac{1}{z} d z
$$

