

DIHEDRAL GROUPS

LECTURE 1, EXERCISE SET 2

Definition 1. The dihedral group D_n ($n \geq 3$) is the group of symmetries of a regular n -sided polygon.

Exercise 2. In this exercise we shall study the group D_3 .

- (1) List all symmetries of an equilateral triangle, giving them “letter” names. Count the number of symmetries. Classify which symmetries are orientation-preserving, and which are orientation-reversing.
- (2) Compute the multiplication table for the group D_3 .

Look at your multiplication table and convince yourself that D_3 is a NON-ABELIAN group. This is the smallest non-abelian group, which also goes by the name S_3 .

Definition 3. A group is called *finite* if it has a finite number of elements. The order of a finite group is the number of elements in the group.

Definition 4. (Informal) We say that a group is generated by two elements x, y if any element of the group can be written as a product of x ’s and y ’s.

Exercise 5. Show that D_3 is generated by 2 elements: x , the rotation by $2\pi/3$ and y , the reflection through the median.

Exercise 6. We shall now investigate the group D_4 , the group of symmetries of a square

- (1) Find the order of the group D_4 .
- (2) Find two symmetries of a square such that all other symmetries can be obtained by consecutive compositions of these two. Write down every symmetry as a composition of the two you have chosen. Once you are done, you’ve established that D_4 is generated by 2 elements! The two chosen symmetries are the *generators* of D_4 .