DIHEDRAL GROUPS

LECTURE 2, EXERCISE SET 2: CONTINUE FROM LECTURE 1

Definition 1. The dihedral group D_n ($n \geq 3$) is the group of symmetries of a regular n-sided polygon.

Exercise 2. Finish the exercise about D_3:

(1) List all symmetries of an equilateral triangle, giving them "letter" names. Count the number of symmetries. Classify which symmetries are orientation-preserving, and which are orientation-reversing.
(2) Compute the multiplication table for the group D_3.

Look at your multiplication table and convince yourself that D_3 is a NON-ABELIAN group. This is the smallest non-abelian group, which also goes by the name S_3.

Definition 3. A group is called *finite* if it has a finite number of elements. The order of a finite group is the number of elements in the group.

Definition 4. (Informal) We say that a group is generated by two elements x, y if any element of the group can be written as a product of x's and y's.

More generally, a subset of elements $\{x_1, x_2, \dots\}$ of G is a set of *generators* of a group G if any element of G can be written as a product of elements x_i from the subset.

Exercise 5. Show that D_3 is generated by 2 elements: ρ, the rotation by $2\pi/3$ and r, the reflection through the median.

Exercise 6. We shall now investigate the group D_4, the group of symmetries of a square

(1) Find the order of the group D_4.
(2) Find two symmetries of a square such that all other symmetries can be obtained by consecutive compositions of these two. Write down every symmetry as a composition of the two you have chosen. Once you are done, you've established that D_4 is generated by 2 elements! The two chosen symmetries are the *generators* of D_4.

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