DIHEDRAL GROUPS: GENERATORS AND RELATIONS

LECTURE 2, EXERCISE SET 2

Exercise 1. Consider D_3. Let ρ and r be generators as in the previous exercise set. Redo your multiplication table for D_3 labeling the elements of the group as products of ρ and r (so that every element looks like $\rho^i r^j$).

Don't forget to compose from *right to left*!

Exercise 2. Now do the same thing for D_4. Let ρ be the rotation by $\pi/2$, and r be the reflection through the main diagonal. Compute the multiplication table for D_4 labeling all elements as products of powers of ρ and r.

Definition 1. (Informal) A *relation* on the elements of a group is any equation that the elements satisfy.

Example 2. Group D_3 satisfies, for example, the relations $\rho^3 = e$ and $r^2 = e$.

Definition 3. A *free* group on two generators is a group generated by two elements with no relations between them.

Any group generated by two elements can be obtained from a free group on two generators by imposing some relations on the generators. Such presentation of a group is called "presentation by generators and relations".

Exercise 3.
(1) Find an exhausting family of relations for the two generators of D_3,
(2) Find an exhausting family of relations for the two generators of D_4.

If you have time left, try to generalize to any D_n:

Exercise 4.
(1) Describe a dihedral group D_n with generators and relations.
(2) Let ρ, r be the "standard" generators of D_n (ρ is rotation, r is reflection). Express the element $\rho^2 r \rho^{-1} r^{-1} \rho^3 r^3$ in the form $\rho^i r^j$.

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