## MATRIX MULTIPLICATION

LECTURE 3, EXERCISE SET 1

Exercise 1. In each example below, multiply the two given matrices.
(1) $\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$
(2) $\left[\begin{array}{rr}1 & -1 \\ 1 & 1\end{array}\right]\left[\begin{array}{rr}1 / 2 & 1 / 2 \\ -1 / 2 & 1 / 2\end{array}\right]$
(3) $\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]\left[\begin{array}{ll}\mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d}\end{array}\right]$
(4) $\left[\begin{array}{ll}\mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d}\end{array}\right]\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$
(5) $\left[\begin{array}{ll}\mathrm{t} & 0 \\ 0 & \mathrm{t}\end{array}\right]\left[\begin{array}{ll}\mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d}\end{array}\right]$
(6) $\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]\left[\begin{array}{ll}\mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d}\end{array}\right]\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$

The matrix $\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$ is called a permutation matrix. Why is that :)?
(7) $\left[\begin{array}{ll}\mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d}\end{array}\right]\left[\begin{array}{rr}\mathrm{d} & -\mathrm{b} \\ -\mathrm{c} & \mathrm{a}\end{array}\right]$

This is the end of the first exercise. When you are done, choose one person who may possibly go to the board to write down the answer to one of the exercises. Raise your hand when your entire family (that is, everyone at your table) is finished and a family representative has been determined (via a democratic procedure, of course!).
Exercise 2. Give an example of two matrices $A, B$, such that $A B \neq B A$.
Definition. The matrix $I=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$ is called the identity matrix.
Definition. Let $A=\left[\begin{array}{ll}\mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d}\end{array}\right]$. The determinant of $A$ is $\operatorname{det} A=a d-b c$.
Definition. A matrix $A$ is called invertible if there exists matrix $B$ such that $A B=B A=I$.

Exercise 3. Prove that $A$ is invertible if and only if $\operatorname{det} A \neq 0$.

