ORBITS

LECTURE 6, EXERCISE SET 1

Definition 1. Let G be a group of motions of the plane (that is, G is a subgroup of M), and s be a point. The *orbit* of s is the set of all points into which s can be transformed by the motions in G.

Exercise 2. Plot the orbits for the following points and groups:

(1) For $s = (1, 0)$, and $G = D_3$;
(2) For $s = (1, 2)$, and $G = D_4$;
(3) For $s = (\sqrt{3}, 1)$, $G = D_6$

Theorem 1. *(Fixed Point Theorem.) Let G be a finite group of rigid motions of the plane. There exists a point s on the plane which is fixed under the action of G (that is, any elements of the group G leaves s in place*

Exercise 3. Consider a subgroup of M generated by two rotations: a rotation around the origin by the angle $2\pi/5$, and a rotation around the point $(1, 1)$ by the angle $\pi/3$. Is this group finite or infinite?

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