ADDITIONAL PROBLEMS: GROUP OF ORTHOGONAL MATRICES

SYMMETRY AND ESCHER, LECTURE 5

Definition. A matrix $A = \begin{bmatrix} \mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \end{bmatrix}$ is called *orthogonal* if the following conditions are satisfied: $a^2 + c^2 = b^2 + d^2 = 1$, and $ab + cd = 0$.

Definition. The *transpose* of a matrix $A = \begin{bmatrix} \mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \end{bmatrix}$ is the matrix $A^T = \begin{bmatrix} \mathrm{a} & \mathrm{c} \\ \mathrm{b} & \mathrm{d} \end{bmatrix}$

Exercise 1. Let A be an orthogonal matrix.
(a) Show that $\det A = \pm 1$
(b) Show that A^T is also orthogonal

Exercise 2. Prove that orthogonal matrices form a subgroup of $GL_2(\mathbb{R})$.

Exercise 3. Show that the function $A \mapsto \det A$ determines a *group homomorphism* (= map of groups) from $GL_2(\mathbb{R})$ to $\mathbb{R}^*$.

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