MATRIX MULTIPLICATION

SYMMETRY AND ESCHER, EXERCISE SET 4 (LECTURE 3)

Exercise 1. In each example below, multiply the two given matrices.

(1) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(4) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(5) $\begin{bmatrix} t & 0 \\ 0 & t \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(6) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is called a *permutation* matrix. Why is that :)?

(7) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

This is the end of the first exercise. When you are done, choose one person who may possibly go to the board to write down the answer to one of the exercises. Raise your hand when your entire team (that is, everyone at your table) is finished and a team representative has been determined (via a democratic procedure, preferably).

Exercise 2. Give an example of two matrices A, B, such that $AB \neq BA$.

Definition. The matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called the *identity* matrix.

Definition. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. The *determinant* of A is $\det A = ad - bc$.

Definition. A matrix A is called *invertible* if there exists matrix B such that $AB = BA = I$.

Exercise 3. Prove that A is invertible if and only if $\det A \neq 0$.

Date: July 1, 2009.