

**BACK TO GEOMETRY: MATRICES AS TRANSFORMATIONS
OF THE PLANE.**

SYMMETRY AND ESCHER, EXERCISE SET 5 (LECTURE 4)

Exercise 1. Describe geometrically the “action” of each of the matrices below on the plane (for example, the permutation matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is a reflection through the line $x = y$)

$$(1) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$(2) \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$(3) \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$(4) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$(5) \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

$$(6) \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

This is the end of the first exercise in this set. If you have finished ahead of other groups, try to answer the following questions:

- Which of the motions (1)-(6) are orientation preserving?
- Which are orientation reversing?
- Give a necessary and sufficient condition on the matrix A so that the motion of the plane defined by A is orientation-preserving.

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Exercise 2. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

- (1) Compute $v = Be_2$
- (2) Now compute $Av = A(Be_2)$
- (3) Compute the matrix $C = AB$
- (4) Now compute $Ce_2 = (AB)e_2$
??Did you get the same answer for (2) and (4)??
- (5) Describe geometrically the transformation of the plane defined by the matrix C .
- (6) Now describe geometrically the composition of transformations defined by the matrices A and B .
- (7) Make a conclusion and a conjecture!