RIGID MOTIONS AS MATRIX MULTIPLICATION.

SYMMETRY AND ESCHER, EXERCISE SET 6 (LECTURE 5)

Exercise 1. (a) Write down the matrix which corresponds to a counter-clockwise rotation around the origin by the angle ϕ.

(b) Let B be the matrix corresponding to a counter-clockwise rotation around the origin by the angle ϕ, and A be the matrix corresponding to a counter-clockwise rotation around the origin by the angle ψ. Then AB corresponds to the rotation by ϕ followed by the rotation by ψ which is clearly just a rotation by $\phi + \psi$. Using matrix multiplication, compute AB.

(c) The matrix AB you got using matrix multiplication corresponds to rotation by $\phi + \psi$. Hence, you have just proved the laws of sin and cos!

$$\cos(\phi + \psi) = \cos\phi\cos\psi - \sin\phi\sin\psi$$
$$\sin(\phi + \psi) = \cos\phi\sin\psi + \cos\psi\sin\phi$$

Problem 2. Write down the matrix which corresponds to the reflection through the line $y = ax$.

Problem 3. (a) Describe all rigid motions that correspond to multiplication by a matrix

(b) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and assume A defines a rigid motion (that is, multiplication by A preserves distances). Clearly, not every matrix satisfies this property. What restrictions/conditions does this put on a, b, c, d? Treat this as an open-ended question: list the conditions you can come up with; convince the other members of your group that your conditions are necessary. When you exhaust your resources, try to prove your list of conditions is sufficient.

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