

DIHEDRAL GROUP D_n

LECTURE 2, EXERCISE SET 2
PROBLEM SET III

Definition. The dihedral group D_n ($n \geq 3$) is the group of symmetries of a regular n -sided polygon.

Let $\rho = \rho_{\frac{2\pi}{n}}$ be the rotation counter-clockwise by $\frac{2\pi}{n}$ around the center of the polygon. Let r be a reflection of the polygon (for convenience, let's arrange the coordinate axes so that the center of the polygon is at the origin, and r is the reflection through the x -axis.)

Exercise 1. Prove that any other reflection in D_n has the form $\text{Rot} \circ r = \rho^i \circ r$ for some integer i .

Exercise 2. Let $n = 4$, so that D_4 is the group of symmetries of a square. Simplify: $(\rho \circ r) \circ \rho^2$.

Exercise 3. For any D_n , prove that the following relation holds: $r\rho = \rho^{n-1}r = \rho^{-1}r$.

Exercise 4. (Just for fun...) Simplify: $\rho^2 \circ r \circ \rho^{-1} \circ r^{-1} \circ \rho^3 \circ r^3$.