ORBITS

LECTURE 5, EXERCISE SET 1

Start by proving the "Fixed Point Theorem" from last time (don't forget the lemma about the center of gravity!)

Theorem 1. *(Fixed Point Theorem.) Let G be a finite group of rigid motions of the plane. There exists a point P on the plane which is fixed under the action of G (that is, any elements of the group G leaves P in place).*

Definition 1. Let G be a group of motions of the plane (that is, G is a subgroup of $\mathbb{M}$), and s be a point. The *orbit* of s is the set of all points into which s can be transformed by the motions in G.

Exercise 2. Plot the orbits for the following points and groups:
(1) For $s = (1, 0)$, and $G = D_3$;
(2) For $s = (1, 2)$, and $G = D_4$;
(3) For $s = (\sqrt{3}, 1)$, $G = D_6$

If you are adventurous or like drawing, take a small (preferrable completely asymetric) object instead of a point for the previous exercise, and do the same thing. You are basically playing the game of "kaleidoscope" on paper. Note that the bigger is your group, the more interesting is the end result.

Exercise 3. Consider a subgroup of $\mathbb{M}$ generated by two rotations: a rotation around the origin by the angle $2\pi/5$, and a rotation around the point $(1, 1)$ by the angle $\pi/3$. Is this group finite or infinite? Is it discrete?

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