# CRYSTALLOGRAPHIC RESTRICTION

SYMMETRY AND ESCHER, PROBLEM SET 8 (LECTURE 6)

We now prove the theorem known as *crystallographic restriction*.

**Problem 1.** Let $H < \mathbb{O}$ be a finite subgroup of the group of symmetries of a lattice $L$. Then

(a) Every rotation in $H$ has order $1, 2, 3, 4,$ or $6$.
(b) $H$ is one of the groups $C_n$ or $D_n$ for $n = 1, 2, 3, 4,$ or $6$.

If $G$ is a crystallographic group, and $L$ is the lattice of $G$, then $\overline{G} = G/L$ is the point group. The group $\overline{G}$ consists only of rotations and reflections, and is a finite subgroup of $\mathbb{O}$. It also carries the lattice $L$ to itself. Hence, we have the following important corollary:

**Corollary 2.** *Let $G$ be a 2-dimensional crystallographic group; that is, $G$ is a group of symmetries of a wallpaper pattern. Then the choice for the point group of $G$ (the group which "encodes" all rotations, reflections and glide reflections) is very limited: it is one of the eight (only!!) groups from the list in the last problem.*

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