Definition. The dihedral group $D_n$ ($n \geq 3$) is the group of symmetries of a regular $n$-sided polygon.

Exercise 1. In this exercise we shall study the group $D_3$.

(1) List all symmetries of an equilateral triangle, giving them “letter” names. For example, you can call counter-clockwise rotation by $120^\circ$ by $\rho_{2\pi/3}$. Count the number of symmetries. Classify which symmetries are orientation-preserving, and which are orientation-reversing.

(2) What is the order of $D_3$?

(3) Compute the multiplication table for $D_3$.

Look at your multiplication table and convince yourself that $D_3$ is a NON-ABELIAN group. Do you know another name for $D_3$?

Definition. (Informal) We say that a group is generated by two elements $x$, $y$ if any element of the group can be written as a product of $x$’s and $y$’s.

Exercise 2. Show that $D_3$ is generated by 2 elements: $\rho$, the rotation by $2\pi/3$, and $r$, the reflection through the median.

If you are done with $D_3$ and have time left, think about $D_4$:

Exercise 3. We shall now investigate the group $D_4$, the group of symmetries of a square.

(1) Find the order of the group $D_4$.

(2) Find two symmetries of a square such that all other symmetries can be obtained by consecutive compositions of these two. Write down every symmetry as a composition of the two you have chosen. Once you are done, you’ve established that $D_4$ is generated by 2 elements! The two chosen symmetries are the generators of $D_4$.

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1