Solutions to Math 126A Quiz 4

1. Consider a parametric curve on the plane given by the equations $(x(t), y(t)) = (17 + t^2, t^2 + t^3)$.

   (a) Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ as functions of t.
   Solution: First we compute

$$\frac{dx}{dt} = 2t, \frac{dy}{dt} = 2t + 3t^2.$$

   Now apply the formulas for the first and second derivatives of parametric equations:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t + 3t^2}{2t} = 1 + \frac{3}{2}t.$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\frac{dy}{dx}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}(1 + \frac{3}{2}t)}{2t} = \frac{3}{4t}.$$

   (b) Find all values of t for which the curve is concave upward.
   Solution: Recall from Math 124 that the graph of a function is concave up on an interval I whenever $\frac{d^2y}{dx^2} \geq 0$ on I. From (a), this means the curve is concave up whenever

$$\frac{3}{4t} \geq 0.$$

   But this occurs as long as $t > 0$.

   (c) Set up, but do not evaluate, the integral to compute the length of the arc of the curve between $t = 0$, and $t = 1$.
   Solution:

$$\begin{aligned} L &= \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \\ &= \int_0^1 \sqrt{(2t)^2 + (2t + 3t^2)^2} dt \\ &= \int_0^1 \sqrt{8t^2 + 12t^3 + 9t^4} dt \\ &= \int_0^1 t\sqrt{8 + 12t + 9t^2} dt. \end{aligned}$$