NAME ______________________________ SECTION __________

Practice problems for Midterm I

Math 126, Sections C, D

October, 2006

For the actual midterm you may use one 8×11.5 sheet of *handwritten* notes. You may use your "simple" scientific calculator on the exam. No books, printed notes or graphing calculators. You have to show ALL YOUR WORK to get full credit.

1. (a) Find the quadratic approximation for the function $f(x) = \ln(x)$ based at e.

   (b) Use the polynomial from (a) to estimate $\ln(3)$. Compute the error bound for your approximation. Give your answers in both exact and decimal forms.

2. Find Taylor series and the interval of convergence for the following functions

   (a) $f(x) = \frac{1}{(1-x)^3}$ at $b = 0$

   (b) $f(x) = \frac{x^2-3x-4}{(2x-3)(x^2+4)}$ at $b = 0$

   (c) $f(x) = e^{3x-2}$ at $b = 2$

   (d) $f(x) = 1 - 6x + 2x^{17} - x^{90}$ at $b = 0$.

   (e) $f(x) = \cos^2(x)$ at $b = 0$

   (f) $f(x) = \cos(x^2)$ at $b = 0$

3. Let $f(x) = x^2 \ln(1 + x^3)$.

   (a) Find the Taylor series for $f(x)$ based at $b = 0$.

   (b) Explicitly compute the coefficient by x^{17} in the Taylor expansion from (a).

   (c) Find $f^{17}(0)$.

4. Find the fourth Taylor polynomial based at $b = 0$ of the function $f(x) = \frac{e^{x^2}}{x^2-1}$.

5. Approximate the intgeral $\int_0^2 \sin(x^2)dx$ using the first three non-zero terms of the Taylor series.

6. Let $\bar{u} = (3, 2, -1)$, $\bar{v} = (2, -2, 2)$. Find a unit vector $\bar{w}$ perpendicular to both $\bar{u}$ and $\bar{v}$.

7. (a) Check if the points $A = (5, 1, 3)$, $B = (7, 9, -1)$, $C = (1, -15, 11)$ are colinear in TWO different ways.

   (b) Now let $A = (1, 2, -3)$, $B = (3, 4, -2)$, $C = (3, -2, 1)$. Check whether the triangle ABC has an obtuse angle. Find the area of the triangle.

8. (a) Show that the equation

$$x^2 + y^2 + z^2 = 4x + z$$

   represents a sphere, and find its center and radius.

   (b) Check that the point $A = (4, 0, 1)$ is on the sphere from (a). Then show that the point $B = (3, 2, 5)$ belongs to the plane tangent to the sphere at the point A.

   Hint: Tangent plane to the sphere at the point A is perpendicular to the radius connecting the center to the point A.

9. (Bonus). Express x^{17} as a sum of powers of $(x - 1)$.