MATH **126** Quiz **Oct. 3** **Name:**

Solutions

1. (a)[1pt] Find linear approximation to the function $f(x) = \arctan x$ at the point $a = 1$.

Recall that the linear approximation to a function f **at a point** a **is given by** $y = f(a) + f'(a)(x - a)$**. Here** $f'(x) = \frac{1}{1+x^2}$ **so** $f'(1) = \frac{1}{2}$ **and** $f(1) = \arctan 1 = \frac{\pi}{4}$**. The linear approximation is** $y = \frac{\pi}{4} + \frac{1}{2}(x - 1)$**.**

(b)[1pt] Estimate arctan(1.1). Leave you answer in exact form.

We use the linear approximation at the point $a = 1$ **as above: we plug in** $x = 1.1$ **and the resulting value of** y **should be approximately equal to** $\arctan 1.1$**. This is** $\frac{\pi}{4} + \frac{1}{2}(1.1 - 1) = \frac{\pi}{4} + \frac{1}{2}(0.1) = \frac{\pi}{4} + \frac{1}{20}$**.**

2. Evaluate the following integrals

(a)[2pt] $\int \frac{xdx}{1-x^2}$

First let's try the substitution $u = x^2$, $du = 2xdx$**.**

$\int \frac{xdx}{1-x^2} = \int \frac{du}{2(1-u)}$**.**

Now let's substitute $v = 1 - u$, $dv = -du$**.**

$\int \frac{du}{2(1-u)} = \frac{-1}{2} \int \frac{dv}{v} = \frac{-1}{2} \ln v + C = \frac{-1}{2} \ln(1-u) + C = \frac{-1}{2} \ln(1-x^2) + C$, **where** C **is an arbitrary constant of integration. We could simplify this further if we wished.**

(b)[2pt] $\int_0^{\frac{3\pi}{2}} \sin x \cos x \, dx$

We use the substitution $u = \sin x$, $du = \cos x \, dx$**.**

$\int_0^{\frac{3\pi}{2}} \sin x \cos x \, dx = \int_0^{-1} u \, du = \frac{1}{2}(1 - 0) = \frac{1}{2}$**.**

(c)[2pt] $\int x^2 e^x \, dx$

We use integration by parts. Recall that $\int u \, dv = uv - \int v \, du$**. First we take** $u = x^2$, $dv = e^x \, dx$ **so, differentiating and integrating,** $du = 2x \, dx$, $v = e^x$**. Thus,** $\int x^2 e^x \, dx = x^2 e^x - 2 \int x e^x \, dx$**.**

Now we use integration by parts again on this. Here we take $u = x$, $dv = e^x \, dx$ **so** $du = dx$ **and** $v = e^x$**.** $\int x e^x \, dx = x e^x - \int e^x \, dx = x e^x - e^x + C$, **where** C **is an arbitrary constant of integration.**

Putting this all together, $\int x^2 e^x \, dx = x^2 e^x - 2x e^x + 2e^x + C$**.**

(d)[2 Bonus points!] $\int_{-1}^{0} \frac{dx}{x^2 + 2x + 2}$

We shall simplify our denominator by completing the square. Observe that $x^2+2x+2 = (x^2+2x+1)+1 = (x+1)^2+1$**. Now substitute** $u = x+1$**.**

$$\int_{-1}^{0} \frac{dx}{x^2+2x+2} = \int_{-1}^{0} \frac{dx}{(x+1)^2+1} = \int_{0}^{1} \frac{du}{u^2+1} = \arctan 1 - \arctan 0 = \frac{\pi}{4}.$$