

No books, notes or graphing calculators. Please turn off your cell phones. Show ALL your work.

1. (a) Find the Taylor series for the function  $f(x) = \ln(1+x^2)$  based at  $a = 0$ .

$$\frac{1}{1-x} = \sum x^k$$

$$-\ln(1-x) = \int \frac{1}{1-x} = \int \sum_{k=0}^{\infty} x^k = \sum_{k=0}^{\infty} \int x^k = \sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1} = \sum_{k=1}^{\infty} \frac{x^k}{k}$$

$$\text{so } \ln(1-x) = - \sum_{k=1}^{\infty} \frac{x^k}{k}$$

$$\ln(1+x^2) = \ln(1-(-x^2)) = - \sum_{k=1}^{\infty} \frac{(-x^2)^k}{k} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^{2k}}{k}$$

- (b) Using quadratic approximation for the function  $f(x)$  above, estimate  $\ln(5/4)$ . Leave your answer in exact form.

$$\text{From a) } \ln(1+x^2) = \frac{x^2}{1} + \frac{-1}{2}x^4 + \dots$$

quadratic approximation is  $x^2$

$$\ln(5/4) = \ln(1+(1/2)^2)$$

so use  $x=1/2$

$$\ln(5/4) \approx (1/2)^2 \approx \boxed{1/4}$$