MATH **126** Quiz **Oct. 24** **Name:**

No books, notes or graphing calcuators. Turn off your cell phones.

(5) 1. Determine whether the lines

$$L_1 : \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1} \quad \text{and} \quad L_2 : \frac{x-2}{2} = \frac{y-6}{-1} = \frac{z+2}{3}$$

are parallel, skew or intersecting. If they intersect, find the point of intersection.

Solution. Directional vectors of these lines are $\mathbf{v}_1 = (2, 2, -1)$ and $\mathbf{v}_2 = (2, -1, 3)$. Since they are not parallel, lines L_1 and L_2 are not parallel as well.

Suppose L_1 and L_2 are intersecting at some point (x, y, z). From the equation of L_1 we have $y = x + 2$ and $z = \dfrac{5-x}{2}$. From the equation of L_2 we have $y = \dfrac{14-x}{2}$ and $z = \dfrac{3x-10}{2}$. At the point of intersection we must have $y = x + 2 = \dfrac{14-x}{2}$ and $z = \dfrac{5-x}{2} = \dfrac{3x-10}{2}$. Solving the first of these equations we get $x = \dfrac{10}{3}$, while the solution of the second one is $x = \dfrac{15}{4}$. Therefore, lines L_1 and L_2 do not have a point in common, thus they are skew.

(5) 2. Find an equation of the plane through the origin, and the points $(2, -4, 6)$ and $(5, 1, 3)$.

Solution. Since this plane contains points $(0, 0, 0)$, $(2, -4, 6)$ and $(5, 1, 3)$, it is parallel to vectors $\mathbf{a} = (2, -4, 6)$ and $\mathbf{b} = (5, 1, 3)$. Then a normal vector is given by the cross-product of these:

$$\mathbf{n} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 6 \\ 5 & 1 & 3 \end{vmatrix} = -18\mathbf{i} + 24\mathbf{j} + 22\mathbf{k}.$$

By taking the origin as a point on the plain, we get the equation $-18x + 24y + 22z = 0$.