MATH **126** Quiz **Nov. 7,** *Solutions* **Name:**

(5) 1. Find the Cartesian equation of the curve given by the following equation in polar coordinates:

$$r = 2\sin\theta$$

Sketch the curve.

Let us multiply both sides of the equation by r, to obtain $r^2 = 2r\sin\theta$. This converts easily into Cartesian coordinates as $x^2+y^2 = 2y$ or $x^2+y^2-2y = 0$. Completing the square, we have $x^2+y^2-2y+1 = 1$ or $x^2 + (y-1)^2 = 1$.

This is a circle of radius 1 centered at the point $(0,1)$.

(5) 2. Consider the helix given by the vector equation $\mathrm{r}(t) = (\cos t, \sin t, t)$

(3pt) (a) Find the length of the arc of the helix between the $t = 0$ and $t = 2\pi$ (one revolution).

The arc length function is $s(t) = \int_0^t \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt = \int_0^t \sqrt{\sin^2 t + \cos^2 t + 1}\,dt = \int_0^t \sqrt{2}\,dt = \sqrt{2}t$. The length of one revolution of the helix is then $s(2\pi) = 2\sqrt{2}\pi$.

(2pt) (b) Is parameterization $\mathrm{r(t) = (\cos t, \sin t, t)}$ a *natural* parameterization of the helix? If not, give the natural parametrization.

A natural parametrization is a parametrization by arc length, a parametrization for which $s(t) = t$ or equivalently for which the speed is always 1. From the above, we see that our parametrization has constant speed $\sqrt{2}$, so it is not natural. We can "naturalize" it by solving for $t = \frac{1}{\sqrt{2}}s$, and taking the parametrization $\mathrm{r}(s) = (\cos(\frac{1}{\sqrt{2}}s), \sin(\frac{1}{\sqrt{2}}s), \frac{1}{\sqrt{2}}s)$.