MATH 126 Quiz Nov. 14 KEY Name:

No books, notes or graphing calculators. Please turn off your cell phones. Show ALL your work.

This quiz is TWO-SIDED!

1. Let $f(x,y) = \sqrt{x^2 - y}$

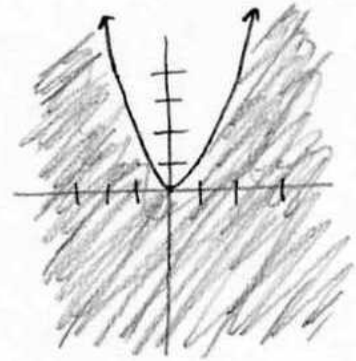
(a) Find and sketch the domain of f.

$$x^2 - y \geq 0$$

$$\Updownarrow$$

$$y \leq x^2$$

$$\text{Domain} = \{ (x,y) : y \leq x^2 \}$$



(b) What is the range of f?

$\sqrt{x^2 - y} \geq 0$ for any (x,y)

$$\text{Range} = [0, \infty)$$

(c) Sketch the level curves for f at 0, 1 and 2.

$$\sqrt{x^2 - y} = c$$

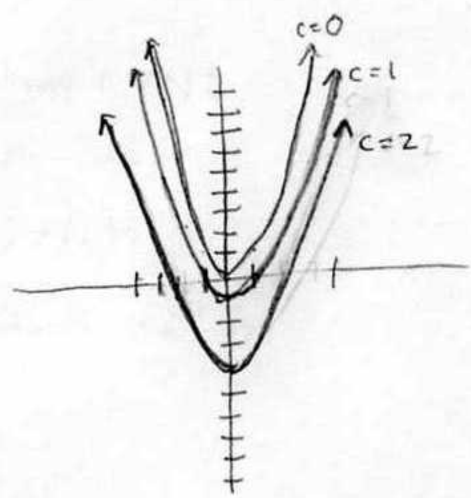
$$x^2 - y = c^2$$

$$y = x^2 - c^2$$

$c = 0$ $y = x^2$

$c = 1$ $y = x^2 - 1$

$c = 2$ $y = x^2 - 4$



2.

Find the velocity and position vectors of a particle moving with the constant acceleration given by the vector

$$a(t) = (0, 0, -10)$$

with the initial velocity $v(0) = (1, 1, -1)$ and initial position $r(0) = (2, 3, 0)$.

$a(t) = (0, 0, -10)$

$v(t) = (c_1, c_2, -10t + c_3)$

$v(0) = (1, 1, -1) \Rightarrow c_1 = 1,\ c_2 = 1,\ c_3 = -1$

$\boxed{v(t) = (1, 1, -10t - 1)}$

$r(t) = (t + d_1, t + d_2, -5t^2 - t + d_3)$

$r(0) = (2, 3, 0) \Rightarrow d_1 = 2,\ d_2 = 3,\ d_3 = 0$

$\boxed{r(t) = (t + 2, t + 3, -5t^2 - t)}$

Find an equation of the osculating plane to the curve given by the position function that you found in (a) at the point $t = 1$.

$r(t) = (t + 2, t + 3, -5t^2 - t)$

$r'(t) = v(t) = (1, 1, -10t - 1)$

$r''(t) = a(t) = (0, 0, -10)$

$B(t)$ is parallel to $r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 1 & 1 & -10t-1 \\ 0 & 0 & -10 \end{vmatrix}$

$= -10i + 10j$

$B(1)$ is parallel to $(-10, 10, 0)$

and is normal to the osculating plane.

$r(1) = (3, 4, -6)$

So an equation for the osculating plane at $t = 1$ is:

$\boxed{-10(x - 3) + 10(y - 4) + 0(z + 6) = 0}$

or

$\boxed{x - y + 1 = 0}$