Homework 4 for 506, Spring 2016
due Friday, May 13

Throughout this homework, k will be a field.

Problem 1. Let $V \subset \mathbb{A}_k^n, W \subset A_k^m$ be algebraic sets.

(1) Show that $f : V \to W$ is a regular morphism if and only if there exists $F_1(x_1, \ldots, x_n), \ldots, F_m(x_1, \ldots, x_n) \in k[x_1, \ldots, x_n]$ such that for any $v \in V$,
$$fv) = (F_1(v), \ldots, F_m(v)).$$
(2) Prove that a regular morphism $f : V \to W$ is continuous in Zariski topology.

Problem 2. Let V, W be affine algebraic varieties. For $f : V \to W$, a regular morphism, denote by $f^* : k[W] \to k[V]$ the map of algebras defined by the formula
$$f^*(\phi) = \phi \circ f$$
for any $\phi \in k[W]$.

(1) Show that f^* is an algebra homomorphism.
(2) Show that the correspondence $f \mapsto f^*$ defines a bijection between the set of regular morphisms between V and W and algebra homomorphisms between $k[W]$ and $k[V]$.
(3) Show that f is a regular isomorphism of varieties if and only if f^* is an algebra isomorphism.

Problem 3. Describe irreducible components of the following algebraic sets in $\mathbb{A}_k^3$:

(1) $V((f, g, h))$ where $f = y^2 - xz, g = x^4 - yz, h = z^2 - x^3y$.
(2) $V((xz - y^2, z^3 - x^5))$