Homework III for Group Cohomology, Winter 2006
due Monday, March 13, before noon

Problem 1. Let H be a normal subgroup of G. All modules are considered over $\mathbb{Z}$. Verify that the functors

$$\mathcal{G} = (-)^H : G - mod \to G/H - mod \text{ and } \mathcal{F} = (-)^{G/H} : G/H - mod \to \mathbb{Z} - mod$$

satisfy the conditions necessary to construct the Grothendieck spectral sequence

$$H^p(G/H, H^q(H, M)) \Rightarrow H^{p+q}(G, M)$$

.

Problem 2. Let m be an odd integer, and let $D_{2m} = C_m \rtimes C_2$ be the dihedral group. Compute $H^*(D_{2m}, \mathbb{Z})$.

Hint. To apply Lyndon-Hochschild-Serre spectral sequence $H^p(C_2, H^q(C_m, \mathbb{Z})) \Rightarrow H^{p+q}(D_{2m}, \mathbb{Z})$ you will need to compute the action of C_2 on $H^q(C_m, \mathbb{Z})$. You can use the explicit map on periodic resolutions constructed in Example 6.7.10 in Weibel to compute this action. This should give you the E_2 page of the spectral sequence. Carefully tracking down the numbers, you shall notice that it has no non-trivial differentials, and, thus, gives you E_∞ right away.