MATH 341 Lin Alg I Fall 2003
Practice Midterm **Name:** ____________________

1. Let $A = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$

   1. Is A invertible? If so, find A^{-1} in two different ways.
   2. Let T be the linear transformation of the plane $\mathbf{R}^2$ defined by the matrix A. Is T onto? Is it one-to-one?
   3. Draw a picture of the triangle on the plane with coordinates (-1,2), (1,1), (2,3). Compute the area of the trianlge.
   4. Is the system $Ax = b$ consistent for any 2-dimensional vector b. Justify your answer.
   5. Solve
   $$\begin{cases} 3x + 5y &= 4 \\ 2x + 4y &= 1000 \end{cases}$$
   using Cramer's rule. Check your answer by doing row reduction.

2. Check for linear inependence

   1. $a = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad d = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
   2. $a = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \quad c = \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$
   3. $a = \begin{bmatrix} 1 \\ 2 \\ 5 \\ 0 \end{bmatrix}, \quad b = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad c = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 0 \end{bmatrix}$

3. Find all points in the intersection of the planes in $\mathbf{R}^3$ given by the equations $3x-5y+7z = 5$ and $x - 2y + z = 9$. Describe intersection geomerically. Is it a subspace of $\mathbf{R}^3$?

4. Let $u = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$ and $u = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$. Are u and v linearly independent? What is the dimension of Span $\{u, v\}$? For each of the following vectors, indicate whether or not it lies in Span $\{u, v\}$:
$$a = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad c = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \quad d = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad e = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$

5. Let T be a linear transformation. Define what it means for T to be
   (a) *onto*
   (b) *one-to-one*
   (c) *invertible*

6. Let T be a linear transformation with the standard matrix $A = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix}$ and S be a linear trasnformation with the standard matrix $B = \begin{pmatrix} -1 & 5 & -8 \\ 1 & -2 & 2 \\ -1 & -1 & 1 \end{pmatrix}$. What is the matrix of the composition $S \circ T$? Describe this composition geometrically. Without doing any additional computations, answer the following questions (provide justification):

   (a) Is T onto?

   (b) Is T one-to-one?

   (c) Is S invertible?

7. Let $A = \begin{pmatrix} 0 & -1 & 2 & 18 \\ 1 & 3 & 2 & -3 \\ 0 & 1 & 0 & 2 \\ 1 & 2 & 1 & 0 \end{pmatrix}$

   (a) Compute Det A.

   (b) What is Rank A?

   (c) Do the columns of A span all of $\mathbf{R}^4$?

   (d) Are the columns of A linearly independent?

8. Let $\{v_1, v_2, v_3\}$ be linearly dependent vectors in $\mathbf{R}^4$. If T is a linear transformation of $\mathbf{R}^4$, show that $\{T(v_1), T(v_2), T(v_3)\}$ are linearly dependent vectors.

   Suppose it is given that $\{T(v_1), T(v_2), T(v_3)\}$ are linearly dependent. Is it true or false that $\{v_1, v_2, v_3\}$ are linearly dependent? Justify your answer: give a proof if it is positive, construct a counterexample otherwise.

9. Let $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 5 & 0 & 1 \end{pmatrix}$. Give bases for the column space Col A and the null space Null A, determine the rank and nullity (dim Nul A) of A.